Manonmaniam Sundaranar University,

Directorate of Distance & Continuing Education,

Tirunelveli - 627 012 Tamilnadu, India

OPEN AND DISTANCE LEARNING (ODL) PROGRAMMES

(FOR THOSE WHO JOINED THE PROGRAMMES FROM THE ACADEMIC YEAR 2023–2024)

B.Sc. Physics

I Year

PROPERTIES OF MATTER AND ACOUSTICS

Course Material

Prepared

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Tirunelveli - 12

PROPERTIES OF MATTER AND ACOUSTICS

UNIT I ELASTICITY:

Hooke's law – stress-strain diagram – elastic constants –Poisson's ratio – relation between elastic constants and Poisson's ratio – work done in stretching and twisting a wire – twisting couple on a cylinder – rigidity modulus by static torsion – torsional pendulum (with and without masses)

UNIT II **BENDING OF BEAMS:**

Cantilever– expression for Bending moment – expression for depression at the loaded end of the cantilever– oscillations of a cantilever – expression for time period – experiment to find Young's modulus – non-uniform bending– experiment to determine Young's modulus by Koenig's method – uniform bending – expression for elevation – experiment to determine Young's modulus using microscope.

UNIT III **FLUID DYNAMICS:**

Surface tension: definition – molecular forces– excess pressure over curved surface – application to spherical and cylindrical drops and bubbles – determination of surface tension by Jaegar's method– variation of surface tension with temperature

Viscosity: definition–streamline and turbulent flow–rate of flow of liquid in a capillary tube – Poiseuille's formula –corrections – terminal velocity and Stoke's formula– variation of viscosity with temperature

UNIT IV **WAVES AND OSCILLATIONS:**

Simple Harmonic Motion (SHM) – differential equation of SHM – graphical representation of SHM – composition of two SHM in a straight line and at right angles – Lissajous's figures-free, damped, forced vibrations –resonance and Sharpness of resonance. Laws of transverse vibration in strings –sonometer – determination of AC frequency using sonometer – determination of frequency using Melde's string apparatus

UNIT V **ACOUSTICS OF BUILDINGS AND ULTRASONICS:**

Intensity of sound – decibel – loudness of sound –reverberation – Sabine's reverberation formula – acoustic intensity – factors affecting the acoustics of buildings.

Ultrasonic waves: production of ultrasonic waves – Piezoelectric crystal method – magnetostriction effect – application of ultrasonic waves

UNIT VI **PROFESSIONAL COMPONENTS:** expert lectures –seminars — webinars – industry inputs – social accountability – patriotism

TEXT BOOKS

1. D.S.Mathur, 2010, Elements of Properties of Matter, S.Chand and Co
2. Brij Lal and N.Subrahmanyam, 2003, Properties of Matter,S.Chand and Co
3. D.R.Khanna and R.S.Bedi,1969,Textbook of Sound, Atma Ram and sons
4. Brij Lal and N.Subrahmanyam,1995, A Text Book of Sound, Second revised edition, Vikas Publishing House.
5. R.Murugesan, 2012, Properties of Matter,S.Chand and Co.

Unit 1: Elasticity

Hooke's law – stress - strain diagram – elastic constants –Poisson's ratio – relation between elastic constants and Poisson's ratio – work done in stretching and twisting a wire – twisting couple on a cylinder – rigidity modulus by static torsion – torsional pendulum (with and without masses)

1.1 Introduction:

When an external force is applied to a rigid body, there is a change in its length, volume (or) shape. When external forces are removed, the body tends to regain its original shape and size. Such a property of a body by virtue of which a body tends to regain its original shape (or) size when external forces are removed is called elasticity.

The SI unit for elasticity is the Pascal (Pa). It is defined as force per unit area. Typically, it is a measure of pressure, which in classical mechanics points to stress. The Pascal has the dimension $L^{-1}\cdot M\cdot T^{-2}$.

1.2 Hooke's Law:

Stress and strain take different forms in different situations. Generally, for small deformations, the stress and strain are proportional to each other, and this is known as Hooke's Law.

Hooke's law states that the strain of the material is proportional to the applied stress within the elastic limit of that material.

When the elastic materials are stretched, the atoms and molecules deform until stress is applied, and when the stress is removed, they return to their initial state.

Mathematically, Hooke's law is expressed as:

$$F = -kx$$

In the equation, F is the force, x is the extension in length, k is the constant of proportionality known as the spring constant in N/m.

1.3 Stress - Strain Diagram

***Stress**:* When the body is deformed by the application of external forces, the forces within the body are brought into play. Elastic bodies regain their original shape due to internal restoring forces. The internal forces and external forces are opposite in direction. If a force F is applied uniformly over a surface of area A, then the stress is defined as the force per unit area. Thus, Unit for stress is Nm^{-2}

$$Stress = \frac{Force}{Area}$$

***Strain*:** A body under stress gets deformed. The fractional change in the dimension of a body produced by the external stress acting on it is called strain. The ratio of charge of any dimension to its original dimension is called strain. Since strain is the ratio of two identical physical quantities, it is just a number. It has no unit or dimension.

$$\boldsymbol{Strain = \frac{Change\ in\ dimension}{Initial\ dimension}}$$

***Stress-Strain curve**:* From the origin till the proportional limit nearing yield strength, the straight line implies that the material follows Hooke's law. Beyond the elastic limit between proportional limit and yield strength, the material loses its elasticity and exhibits plasticity. The area under the curve from origin to the proportional limit falls under the elastic range. The area under the curve from a proportional limit to the rupture/fracture point falls under the plastic range.

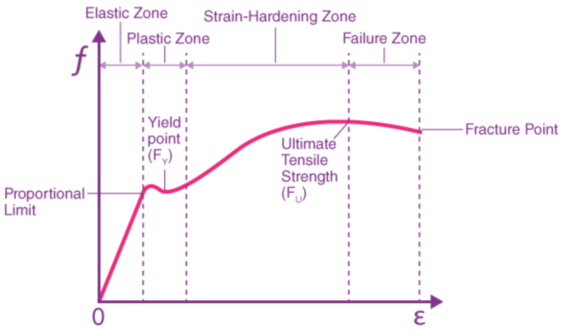


Figure 1. 1

The material's ultimate strength is defined based on the maximum ordinate value given by the stress-strain curve (from origin to rupture). The value provides the rupture with strength at a point of rupture.

Applications:

- It is used as a fundamental principle behind the manometer, spring scale, and the balance wheel of the clock.
- Hooke's law sets the foundation for seismology, acoustics and molecular mechanics.

Disadvantages:

- Hooke's law ceases to apply past the elastic limit of a material.
- Hooke's law is accurate only for solid bodies if the forces and deformations are small.
- Hooke's law isn't a universal principle and only applies to the materials as long as they aren't stretched way past their capacity.

1.4 Elastic constants:

Elastic constants are the constants which describe the mechanical response of an elastic material when it is subjected to different kinds of loads. Based on the type of stress and strain, Elastic constants can be classified into 4 types. These Elastic constants are mentioned below.

- Young's Modulus of elasticity (E)
- Bulk Modulus (K)
- Modulus of rigidity (G)
- Poisson's ratio (μ)

Young's Modulus of Elasticity (σ)

Young's modulus of elasticity is an elastic constant that is defined as the ratio of Longitudinal stress to longitudinal strain. When an axial load P is applied along the bar's longitudinal axis, the bar's length will be increased in the direction of the applied load, and stress (σ) is induced in the bar.

$$\sigma = P/A$$

According to Hooke's law, longitudinal stress is directly proportional to longitudinal strain. Hence, $\sigma \propto \varepsilon$

$$\text{So, } \sigma = E\,\varepsilon$$

$$\text{Thus, } E = \sigma/\varepsilon$$

Bulk Modulus (K)

The bulk modulus of elasticity is an elastic constant showing a material's incompressibility. When a body is subjected to three mutually perpendicular stresses of equal intensity (σ). Then the ratio of direct stress (σ) to the corresponding volumetric strain (ϵv) is defined as the bulk modulus (K) for the material of the body. Which is generally denoted as 'K'. Thus,

$$K = \frac{Direct\ Stress}{Volumetric\ Strain} = \frac{\sigma}{\varepsilon V}$$

Volumetric strain: Volumetric strain is defined as the ratio of change in volume of an elastic body to its initial volume. For an equally stressed body in all three mutually perpendicular directions,

$$Volumetric\ Strain = \varepsilon V = \frac{\Delta V}{V} = \varepsilon x + \varepsilon y + \varepsilon z$$

Modulus of Rigidity / Shear Modulus (G)

Modulus of Rigidity is an elastic constant that measures a deformable body's rigidity. The shear modulus or modulus of rigidity expresses the relation between shear stress and shear strain. Modulus of rigidity can be defined as the ratio of shear stress to shear strain.

$$G = \frac{T}{\varphi}$$

Shear strain (φ): Shear strain is defined as the angular deformation of the body when it is subjected to shear stress.

Poisson's ratio (μ)

Poisson's ratio is an elastic constant which is defined as the ratio of lateral strain to longitudinal strain. Poisson's ratio is a unit less quantity, and it is generally denoted as μ or 1/m.

$$Poisson's\ ratio = \frac{-\ Lateral\ Stain}{Longitudinal\ Strain}$$

Material	Poisson's ratio (μ)
Cork	0
Concrete	0.1 to 0.2
Metals	1/4 to 1/3
Rubber, Clay, Paraffin	0.5 (Behaves like a perfectly elastic material)

Relation between elastic constants

Young's modulus, Bulk modulus and shear modulus all three elastic constants can be interrelated by deriving a relation between them known as the Elastic constant formula. But young's modulus (E) and the Poisson ratio ($\boldsymbol{\nu}$) are known as the independent elastic constants, and they can be obtained by performing the experiments.

The bulk modulus and the shear modulus are dependent constants and they are related to Young's modulus and the Poisson ratio.

The relation between Young's modulus and shear modulus is

$$E = 2G\ (1 + v)\ N/m^2$$

The relation between young's modulus and Bulk modulus is:

$$E = 3K\ (1 - 2v)\ N/m^2$$

Derivation of Relation Between Elastic Constants

Consider the relation between Young's modulus and the shear modulus,

$$E = 2G\ (1 + v) \qquad (1)$$

Where, E - Young's modulus

G - Shear modulus

v - Poisson ratio

From equation (1) the value of the Poisson ratio is:

$$v = \frac{E}{2G} - 1 \qquad (2)$$

We know that the relation between Young's modulus and the Bulk modulus is

$$E = 3K\ (1 - 2v)\ N/m^2 \qquad (3)$$

Where, E - Young's modulus

K - Bulk modulus

v - Poisson ratio

Substituting the value of Poisson ratio from equation (2) in (3) and simplify,

$$E = 3K\ \left(1 - 2\frac{E}{2G} - 1\right)$$

$$E = 3K\ \left(1 - \frac{E}{G} - 2\right)$$

$$E = 3K\ \left(3 - \frac{E}{G}\right)$$

$$E = 9K - \frac{3KE}{G}$$

$$EG + 3KE = 9KG$$

$$E\,(G + 3K) = 9KG$$

$$E = \frac{9KG}{G+3K} \qquad (4)$$

Equation (4) is known as the Elastic constant formula and it gives the Relation between elastic constants.

1.5 Work done in twisting a wire

Consider a cylindrical wire of length L and radius a fixed at its upper end and twisted trough an angle θ by applying a torque at the lower end. If c is the torque per unit angular twist of the wire, then the torque required to produce a twist θ in the wire is

$$C = c\,\theta$$

The work done in twisting the wire through a small angle dθ is,

$$C\,d\theta = c\,\theta\,d\theta.$$

The total work done in twisting the wire through an angle θ

$$W = \int_0^\theta c.\theta\,d\theta$$

$$W = \frac{1}{2}c.\theta^2$$

The work done in twisting a wire is stored up in the wire as potential energy.

1.6 Twisting couple on a cylinder

Consider a cylindrical wire of length L and radius a fixed at its upper end and twisted through an angle θ by applying a torque at the lower end. Consider the cylinder to consist of an infinite number of hollow co-axial cylinders. Consider one such cylinder of radius x and thickness dx. A line such as AB initially parallel to the axis OO' of the cylinder is displaced to the position AB' through an angle ϕ due to the twisting torque. The result of twisting the cylinder is a shear strain.

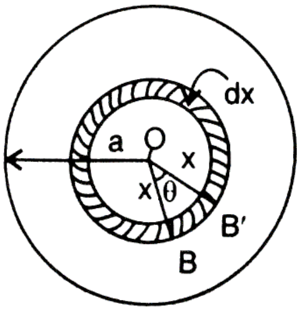
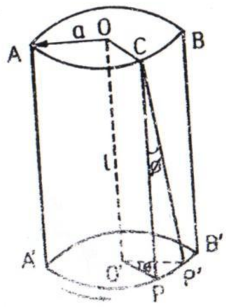


Figure 1. 2



The angle of shear ∠BAB' = ϕ

$$BB' = x.\theta = L\varphi$$

$$\varphi = \frac{x.\theta}{L}$$

We have rigidity modulus $G = \frac{Shearing\ Force}{Angle\ of\ Shear\ (\varphi)}$

Shearing stress G . ϕ = $= \frac{Gx\ \theta}{L}$

Shearing stress $= \frac{Shearing\ force}{Area\ on\ which\ the\ force\ acts}$

Shearing force = Shearing stress × Area on which the force acts

The area over which the shearing force acts = 2 π x dx

Hence, the shearing force $F = \frac{Gx\theta}{L} \times 2\pi x\ dx$

The moment of this force about the axis OO' of the cylinder

$$F = \frac{Gx\theta}{L} 2\pi x\ dx. x = \frac{2\pi G\theta}{L} x^3 dx$$

Twisting torque on the whole cylinder $C = \int_0^a \frac{2\pi G\theta}{L} x^3 dx$

$$C = \frac{\pi G a^4 \theta}{2L}$$

The torque per unit twist (i.e., the torque when θ = 1 radian) $C = \frac{\pi G a^4}{2L}$

Note 1: When an external torque is applied on the cylinder to twist it, at once an internal torque, due to elastic forces, comes into play. In the equilibrium position, these two torques will be equal and opposite.

Note 2: If the material is in the form of a hollow cylinder of internal radius a and external radius b, then

The torque acting on the cylinder $C = \int_a^b \frac{2\pi G\theta}{L} x^3 dx$

$$= \frac{\pi G\theta}{2L}(b^4 - a^4)$$

$$C = \frac{\pi G(b^4 - a^4)}{2L}$$

1.7 Rigidity modulus by Static torsion method

The experimental rod is rigidly fixed at one end A and fitted into the axle of a wheel W at the other end B. The wheel is provided with a grooved edge over which passes a tape. The tape carries a weight hanger at its free end. The rod can be twisted by adding weights to the hanger. The angle of twist can be measured by means of two pointers fixed at Q and R which move over circular scales S1 and S2. the scales are marked in degrees with center zero.

With no weights on the hanger, the initial readings of the pointers on the scales are adjusted to be zero. Loads are added in steps of m kg (conveniently 0.2kg). The readings on the two scales are noted for every load, both while loading and unloading. The experiment is repeated after reversing the twisting torque by winding the tape over the wheel in the opposite way. The observations are tabulated. The readings in the last column give the twist for a load of M kg for the length QR (= L) of the rod.

The radius of the rod and the radius R of the wheel are measured.

If a load of M kg is suspended from the free end of the tape, the twisting torque = MgR

The angle of twist θ degrees = θ . π / 180 radians.

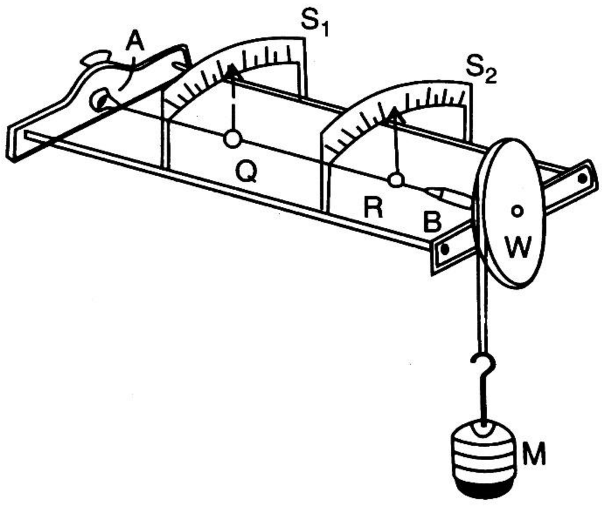


Figure 1. 3

$$\therefore The\ restoring\ torque = \frac{\pi G a^4}{2L} \cdot \frac{\theta \pi}{180}.$$

$$For\ equilibrium, \qquad MgR = \frac{\pi G a^4}{2L} \cdot \frac{\theta \pi}{180}\ or\ G = \frac{360\ M\ g\ R\ L}{\pi^2 a^4 \theta}$$

Since it occurs in the fourth power in the relation used, it should be measured very accurately.

- We eliminate the error due to eccentricity of the wheel by applying the torque in both clockwise and anticlockwise directions
- We eliminate errors due to any slipping at the clamped end by observing readings at two points on the rod.

1.8 Torsional pendulum with masses

The torsion pendulum consists of a wire with one end fixed in a split chuck and the other end to the centre of a circular disc as shown in the figure 1.4. Two equal symmetrical masses (each to m) are placed along a diameter of the disc at equal distances d_1 on either side of the centre of the disc. The disc is rotated through an angle and is then releases. The system executes torsional oscillations about the axis of the wire. The period of oscillations T_1 is determined.

Then

$$T_1 = 2\pi \sqrt{\frac{I_1}{c}}$$

$$T_1^2 = \frac{4\pi^2}{c} I_1$$

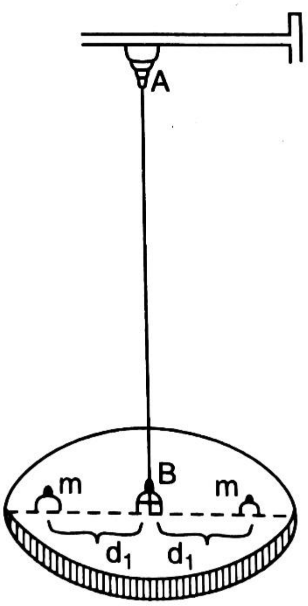


Figure 1. 4

Here,

I_1 = Moment of inertia of the whole system about the axis of the wire

c = torque per unit twist

Let I_0 be the Moment of inertia of the disc alone about the axis of the wire.

I = Moment of inertia of each mass about a parallel axis passing through its centre of gravity.

Then by the parallel axes theorem.

$$I_1 = I_0 + 2i + 2md_1^2$$

$$T_1^2 = \frac{4\pi^2}{c}\left[I_0 + 2i + 2md_1^2\right]$$

The two masses are now kept at equal distances d_2 from the centre of the disc and the corresponding period T_2 is determined. Then,

$$T_2^2 = \frac{4\pi^2}{c}\left[I_0 + 2i + 2md_2^2\right]$$

$$T_2^2 - T_1^2 = \frac{4\pi^2}{c}\,.\,2m\,.\,(d_2^2 - d_1^2)$$

$$c = \frac{\pi G a^4}{2L}$$

$$T_2^2 - T_1^2 = \frac{4\pi^2\ 2m\ (d_2^2 - d_1^2)\ 2L}{\pi G a^4}$$

$$G = \frac{16\pi\ Lm\ (d_2^2 - d_1^2)}{[T_2^2 - T_1^2]a^4}$$

Using this relation, G is determined.

1.9 Torsional pendulum without masses

A torsion pendulum consists of a rigid metallic frame. D is a solid circular disc of a moment of inertia I, mass M and radius R. The wire AB of length l and the radius r is fixed at the end A and the lower end B is clamped to the centre of the disc D as shown in the figure 1.5

When the disc D is rotated about the axis AB, the wire AB gets twisted. The disc executes SHM at any instant, the deflecting couple is equal to the restoring couple.

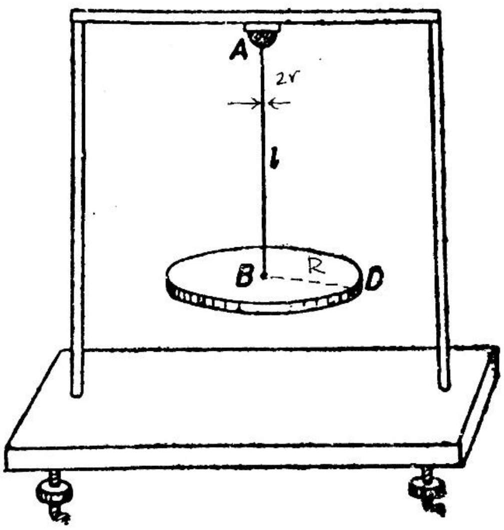


Figure 1. 5

$$I\alpha = -C\theta$$

$$I\frac{d^2\theta}{dt^2} = -C\theta$$

The negative sign shows that the restoring couple is in the opposite direction to the deflecting couple.

$$\frac{d^2\theta}{dt^2} + \left(\frac{C}{I}\right)\theta = 0$$

This is the equation of SHM

$$\omega^2 = \frac{C}{I}$$

$$\omega = \sqrt{\frac{C}{I}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{C}}$$

$$C = \frac{\pi\eta r^4}{2l}$$

$$T = \sqrt{\frac{2Il}{\pi\eta r^4}}$$

$$\eta = \frac{8\pi Il}{T^2r^4}$$

$$I = \frac{MR^2}{2}$$

$$\eta = \frac{4\pi\, MR^2\, l}{T^2r^4}$$

From the above equation η is calculated.

The value of the radius of the wire *AB* should be measured accurately because in the equation it occurs in the fourth power of the radius of the experimental wire

Unit 2: Bending of beams

Cantilever– expression for Bending moment – expression for depression at the loaded end of the cantilever– oscillations of a cantilever – expression for time period – experiment to find Young's modulus – non-uniform bending– experiment to determine Young's modulus by Koenig's method – uniform bending – expression for elevation – experiment to determine Young's modulus using microscope.

2.1 Introduction:

A beam is defined as a road or bar of uniform cross section (circular or rectangular) whose length is very much greater than its thickness. If a beam is fixed at one end and loaded at the other end, it bends. The load acting vertically downwards at its free end and the reaction at the support acting vertically upwards constitute the bending couple, this couple tends to bend the beam clockwise. Since there is no rotation of the beam, the external bending couple must be balanced by another equal and opposite couple which comes into play inside the body due to the elastic nature of the body. The moment of this elastic couple is called the internal bending moment. When the beam is in equilibrium, the external bending moment is always equals to the internal bending moment.

2.2 Expression for the bending moment

Consider a portion of the beam to be bent into a circular arc, as shown in Fig. 1.13. *ef* is the neutral axis. Let *R* be the radius of curvature of the neutral axis and 0 the angle subtended by it at its centre of curvature *C*.

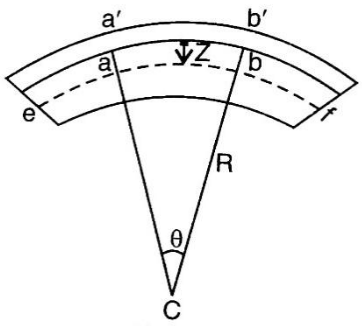


Figure 2. 1

Filaments above *ef* are elongated while filaments below *ef* are compressed. The filament *ef* remains unchanged in length. Let *a'b'* be a filament at a distance *z* from the neutral axis. The length of this filament *a'b'* before bending is equal to that of the corresponding filament on the neutral axis *ab*.

We have original length $ab = R\,\theta.$

Its extended length $a'b' = (R+z)\,\theta$

Increase in its length $a'b' - ab = (R+z)\,\theta - R\,\theta = z.\,\theta$

$$Linear\ Srain = \frac{increase\ in\ length}{original\ length} = \frac{z.\theta}{R.\theta} = \frac{z}{R}$$

If *E* is the young's modulus of the material,

E = Stress / Linear Strain

Stress = E × Linear Strain = E (z/R)

If δA is the area of cross section of the filament,

the tensile force on the area δA = *stress* × *area* $= \frac{E.z}{R}\ \delta A$

Moment of this force about the neutral axis *ef*

$$= \frac{E.z}{R}\ \delta A.z = \frac{E}{R} = \frac{E.z}{R}\delta\ A\ .z^2$$

The sum of the moments of forces acting on all the filaments = $\sum \frac{E}{R}\ \delta A\ z^2$

$$= \frac{E}{R}\sum \delta A\ .z^2$$

$\sum \delta A.z^2$ is called geometrical moment of inertia of the cross section of the beam about an axis through its centre perpendicular to the plane of bending. It is written as equal to Ak^2. i.e.,

$\sum \delta A.z^2 = Ak^2$ (*A* = Area of cross section and *K* = radius of gyration)

But the sum of moments of forces acting on all the filaments is the internal bending moment which comes into play due to elasticity.

Thus, bending moment of a beam = EAk^2/R

- For a rectangular beam of breadth b, and depth (thickness) *d*, $A = bd$ and $k^2 = d^2/12$.

  $Ak^2 = bd^3 / 12$

- For a beam of circular cross section of radius *r*, $A = \pi r^2$ and $k^2 = r^2/4$

  $Ak^2 = \pi r^4 / 4$

- EAk^2 is called the flexural rigidity of the beam.

2.3 Expression for depression at the loaded end of the cantilever

A Cantilever is a beam fixed horizontally at one end and loaded at the other end. Let *OA* be a cantilever of length *l* fixed at *O* and loaded with a weight W at the other end. *OA'* is the unstrained position of the beam. Let the depression *A'A* of the free end be *y* Let us consider an element *PQ* of the beam of length dx at a distance *(QA=x)* from the loaded end. C is the centre of curvature of the element *PQ* and *R* its radius of curvature. The load *W* at *A* and the force of reaction at *W* constitute the external couple, so that, the external bending moment = *W. x*.

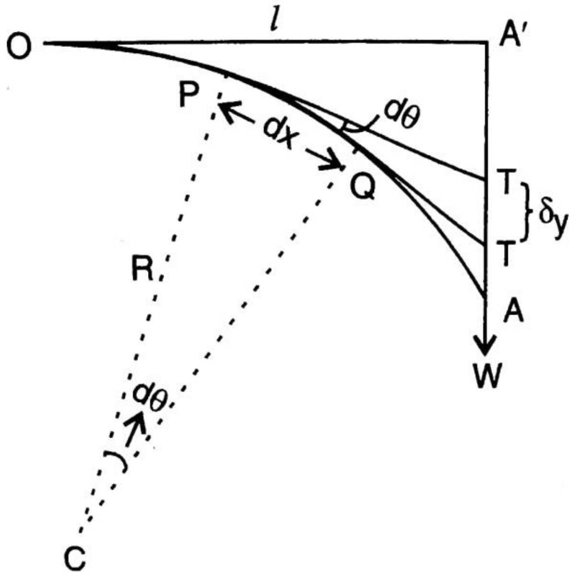
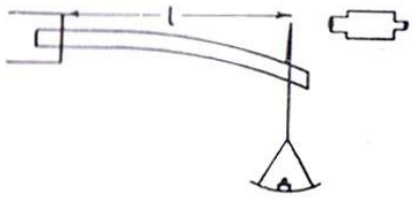


Figure 2. 2



The internal bending moment $= EAk^2$

For equilibrium, $Wx = \frac{EAk^2}{R} or\ R = \frac{EAk^2}{Wx}$

Draw tangents at *P* and *Q* meeting the vertical line at T and S respectively.

Let *TS* = *dy* and *dθ* = Angle between the tangents.

Then, ∠*PCQ* also = dθ

$$\text{Now, } PQ = dx = R\ d\theta$$

$$d\theta = \frac{dx}{R} = dx.\frac{Wx}{EAk^2}$$

We have, $dy = x\ d\theta$

$$= x.\frac{Wx\ dx}{EAk^2} = \frac{Wx^2 dx}{EAk^2}$$

The total depression at the end of the cantilever is

$$y = \int_0^l \frac{Wx^2}{EAk^2} dx$$

$$= \frac{Wl^3}{3\ EAk^2}$$

Angle between the tangents at the end of the cantilever

Since the beam is fixed horizontally at O , the tangent at O is horizontal. If a tangent is drawn at A (which means the free end of the bent bar), it makes angle θ with the horizontal.

Angle between the tangents at *P* and *Q*

$$d\theta = \frac{Wx}{EAk^2}dx$$

Angle between the tangents *O* and *A*

$$\Theta = \int_0^l \frac{Wx}{EAk^2}dx$$

$$= \frac{Wl^2}{2EAk^2}$$

2.4 Oscillations of a cantilever

Let *OA* be a cantilever of length l, of negligible mass fixed at *O*. Let a mass *M* be attached at the other end *A*. If the mass is slightly depressed and then released, the cantilever will execute simple harmonic motion about its original depressed position. The depression of the loaded end of the cantilever is

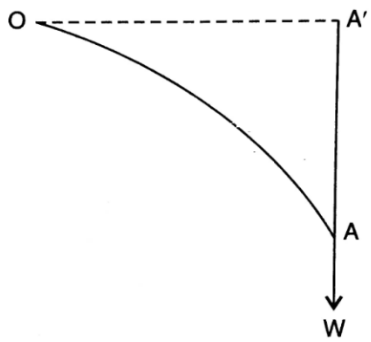


Figure 2. 3

$$y = \frac{Wl^3}{3EAk^2}$$

$$W = \frac{3EAk^2}{l^3}y$$

This must be equal to the elastic reaction of the cantilever balancing it and hence directed opposite to it.

If *M* is the mass of the weight *W* and d^2y/dt^2, the acceleration (upwards), we have

$$\text{Elastic reaction} = M\frac{d^2y}{dt^2}$$

$$-M\frac{d^2y}{dt^2} = \frac{3EAk^2}{l^3}y$$

$$\frac{d^2y}{dt^2} = -\frac{3EAk^2}{Ml^3}y$$

$$\frac{3EAk^2}{l^3} = A\ constant$$

The acceleration of mass M or the free end of the cantilever is thus proportional to its displacement and is directed opposite to it.

It, therefore, executes a S.H.M. of time period T, given by

$$T = 2\pi\sqrt{\frac{Displacement}{Acceleration}}$$

$$= 2\pi\sqrt{\frac{y}{\left(\frac{3EAk^2y}{M.l^3}\right)}} = \sqrt{\frac{Ml^3}{3EAk^2}}$$

If the mass of the cantilever is not negligible, it can be shown that,

$$T = 2\pi\sqrt{\frac{\left(M + \frac{1}{3}m\right)l^3}{3EAk^2}}$$

where m = mass of the cantilever.

The mass of the cantilever can be eliminated by finding the periods T_1 and T_2 for two different masses M_1 and M_2 attached to the cantilever at the same length. Then,

$$T_1^2 = 4\pi^2\frac{\left(M_1 + \frac{1}{3}m\right)l^3}{3\ E\ Ak^2}$$

$$T_2^2 = 4\pi^2\frac{\left(M_2 + \frac{1}{3}m\right)l^3}{3\ E\ Ak^2}$$

$$T_2^2 - T_1^2 = \frac{4\pi^2(M_2 - M_1)l^3}{3EAk^2}$$

$$E = \frac{4\pi^2(M_2 - M_1)l^3}{3Ak^2\ (T_2^2 - T_1^2)}$$

2.5 Experiment for finding Young's modulus Non uniform bending (Koenig's method)

The beam is supported on two knife edges K_1 and K_2 separated by distance l. Two plane mirrors m_1 and m_2 are fixed near the two ends of the beam at equal distances beyond the knife edges. The two plane mirrors face each other, and they are inclined slightly outwards from the vertical.

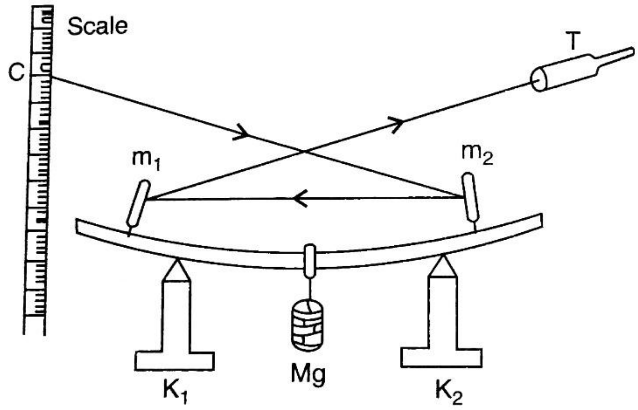


Figure 2. 4

An illuminated translucent scale and a telescope (T) are arranged as shown. The reading of a point C on the scale as reflected first by m_2 and then by m_1 is viewed in the telescope. Let the load suspended at the mid-point of the beam be M. The beam is then bent, and the bending is non uniform. The mirrors at the ends are turned towards each other. Let the shift in the scale reading be s. Young's modulus of the material of the beam is then calculated from the relation

$$E = \frac{3mgl^2(2D + L)}{2bd^3s}$$

l = Distance between the knife edges

D = Distance between the scale and the remote mirror, m_2

L = Distance between the two mirrors

s = Shift in scale reading

b = Breadth of the beam

d = Thickness of the beam

The formula can be deduced as explained below. Let θ be the angle through which each end of the beam has been turned due to loading. Then,

$$\theta = \frac{Wl^2}{16EAk^2}$$

The mirrors m_1 and m_2 also turn through the same angle θ due to loading. In this figure 2.4 m_1 and m_2 represent the initial and m_1' and m_2' the displaced positions of the mirrors. Originally, the image of the scale division at *C* coincides with the cross-wire and finally when the load is applied, H is seen to be in coincidence with the cross-wire. For convenience in evaluating θ, consider the rays of light to be reversed in their path.

TQEC will be the original path. When m_1 is turned through an angle θ to the position m_1', *QE* is turned through 2θ and strikes m_2 at G. Then $EG = L2\theta$. The ray *GH* is turned through an angle 4θ, since, in addition to QE having moved through, 2θ, m_2 itself has turned through θ. Draw *GK* parallel to *EC*. Then, $\angle KGH = 4\theta$ and $CK = EG$. $KH = D\,4\theta$

The total shift in scale reading = s

$$s = EG + KH$$

$$= L\,2\theta + D\,4\theta$$

$$= (L + 2D)\,2\theta$$

$$\theta = \frac{Wl^2}{16EAk^2}$$

$$s = (L + 2D) \times 2 \times \frac{Wl^2}{16\,EAk^2}$$

$$E = \frac{Wl^2(L + 2D)}{8\,Ak^2 s}$$

Now $Ak^2 = bd^3 / 12$ for a beam of rectangular cross section

$W = Mg$.

$$E = \frac{Mgl^2(L + 2D)}{8\left(\frac{bd^3}{12}\right)s} = \frac{3Mgl^2\,(2D + L)}{2bd^3 s}$$

2.6 Expression for elevation experiment to determine Young's modulus using microscope

A beam is supported symmetrically on two knife edges *A* and *B*. Two equal weight hangers are suspended so that their distances from the knife edges are equal. A pin is placed vertically at the centre of the beam. The tip of the pin is viewed by a microscope. The load on each hanger is increased in equal steps of *m* kg and the corresponding microscope readings are noted. similarly, readings are noted while unloading. The results are tabulated as follows:

The mean elevation (*y*) of the centre for *m* kg is found. The length of the beam l between the knife edges and a, the distance between the point of suspension of the load and the nearer knife edge *(AC = CD = a)* are measured. The breadth b and the thickness of *d* of the beam are also measured. The reactions on the knife edges will be *W* and *W*, acting vertically upwards. Consider the cross section of the beam at any point *P*. The only forces acting on the part *PC* of the beam are the forces *W* at *C* and the reaction *W* at *A*.

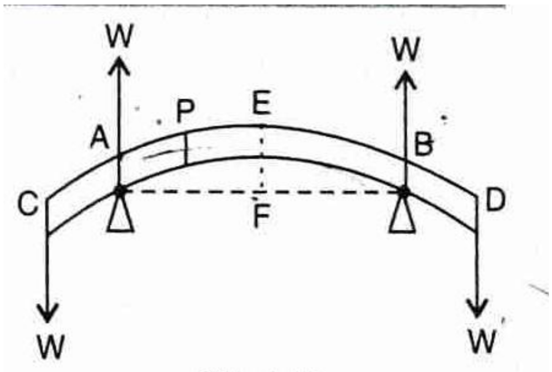


Figure 2. 5

The external bending moment with respect to P

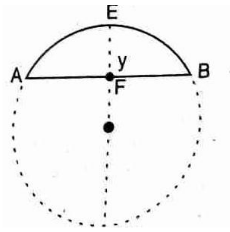
$= W.CP - W.AP = W\,(CP - AP) = W \,.\, AC = Wa.$

This must be balanced by the internal bending moment

EAk^2 / R.

$$Wa = EAk^2 / R \qquad (1)$$

Since for a given load W, E, *a* and Ak^2 are constant, *R* is a constant. The bending is then said to be uniform. If *y* is the elevation of the mid-pint of *AB* above its normal position



$$EF\,(2R - EF) = AF^2$$

$$y\,(2R - y) = (l/2)^2$$

$$y \,.\, 2R = l^2/4$$

$$y = l^2/8R \quad (y^2 \text{ is negligible})$$

From (1),

$$\frac{1}{R} = \frac{Wa}{EAk^2}$$

$$y = \frac{Wal^2}{8EAk^2} = \frac{Mgal^2}{8E\frac{bd^3}{12}}$$

$$W = Mg \text{ and } Ak^2 = bd^3 / 12$$

$$E = \frac{3Mgal^2}{2bd^3y}$$

Using the above formula Young's modulus of the beam can be determined.

Load in kg	*Readings of the scale as seen in the telescope*			*Shift in reading for M kg*
	Load increasing	*Load decreasing*	*Mean*	

Unit 3: Fluid Dynamics

Surface tension: definition – molecular forces– excess pressure over curved surface – application to spherical and cylindrical drops and bubbles – determination of surface tension by Jaegar's method–variation of surface tension with temperature

Viscosity: definition–streamline and turbulent flow–rate of flow of liquid in a capillary tube – Poiseuille's formula –corrections – terminal velocity and Stoke's formula– variation of viscosity with temperature

3.1 Introduction – Surface Tension

Any liquid in small quantity, so that gravity influence is negligibly small, will always assume the form of a spherical drop. e.g., rain drops, small quantities of mercury placed on a clean glass plate etc. So, a liquid must experience some kind of force, so as to occupy a minimum surface area. This contracting tendency of a liquid surface is known as surface tension of liquid. This is a fundamental property of every liquid.

Surface tension is that property of liquids owing to which they tend to acquire minimum surface area. Small liquid drops acquire spherical shape due to surface tension. Big drops flatten due to weight. The following experiment illustrates the tendency of a liquid to decrease its surface area. When a camel hairbrush is dipped into water, the bristles spread out.

When a camel hair brush is dipped into water, the bristles spread out When the brush is taken out, the bristles cling together on account of the films of water between them contracts This experiment clearly shows that the surface of a liquid behaves like an elastic membrane under tension with a tendency to contract. This tension or pull in the surface of a liquid is called its surface tension.

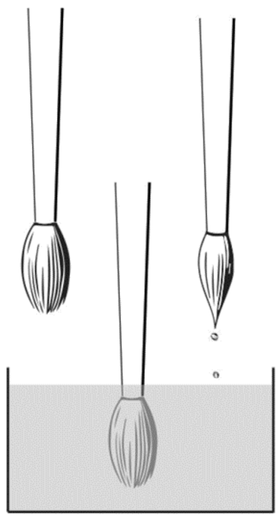


Figure 3. 1

Definition: Surface tension is defined as the force per unit length of a line drawn in the liquid surface, acting perpendicular to it at every point and tending to pull the surface apart along the line.

Unit of Surface Tension: Surface tension is force per unit length. So its SI unit is Newton per meter (Nm^{-1})

3.2 Excess pressure over curved surfaces

A spherical liquid drop has a convex surface. The molecules near the surface of the drop experience a resultant force, acting inwards due to surface tension. Therefore, the pressure inside the drop must be greater than the pressure outside it. Let this excess pressure inside the liquid drop over the pressure outside it he *p*.

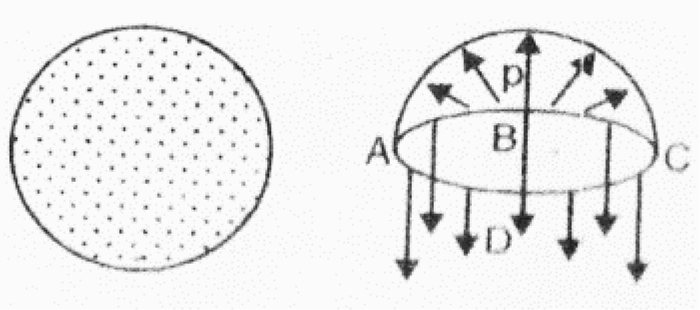


Figure 3. 2

Imagine the drop to be divided into two exactly equal halves. Consider the equilibrium of the upper half of the drop. r is the radius of the drop and σ it's Surface Tension

Upward force on the plane face ABCD due to the excess pressure $p = p\,\pi\, r^2$

Downward force due to surface tension acting along the circumference of the circle $ABCD = \sigma\, 2\, \pi\, r$

$$p\,\pi\, r^2 = \sigma\, 2\, \pi\, r$$

$$p = 2\,\sigma / r$$

3.3 Application to spherical and cylindrical drops and bubbles

A water drop is always in the shape of a sphere although a falling drop may adopt various shapes, because of various forces acting on it. Because of surface tension, the wall tension required for the formation of drops or bubbles is provided. The tendency to minimise surface area, causes the wall tension to be pulled inwards, directed at all sides, thereby leading to a spherical shape.

Cohesive and Adhesive Forces

Cohesive forces are the forces of attraction between molecules of a similar type. For example, the forces of attraction between molecules of water in a glass. Adhesive forces, on the other hand, are forces of attraction between molecules of different types. For example, the force of attraction between water molecules in a glass and the glass molecules.

Capillary Rise

We all know that plants absorb water from the soil to make food (photosynthesis). But have you ever wondered how this happens? For water to rise up, it has to work against gravity and yet it does happen. This is another phenomenon which occurs because of the surface tension of liquids.

If water is placed in a beaker or a narrow measuring cylinder, you can see that the surface of the water meniscus isn't straight. It forms a slight depression. Actually, due to adhesive forces between water and surface, the outer edge is pulled upwards (in case of water). The film formed due to surface tension tends to hold the surface in place and due to this, the entire liquid is pulled upwards, when the edges are pulled upwards.

Say a very thin and long narrow tube is placed in a tub of water, adhesive forces will cause the water to rise a bit in this tube, won't it? When the adhesive forces are greater than the cohesive forces between water molecules, the water tends to rise.

The height to which the water rises is given by the following relation.

$$h = 2\sigma / \rho r g$$

where,

h is the height of rising of liquid due to capillary action

σ is the surface tension of the liquid

ρ is the density of the liquid

g is the acceleration due to gravity

r is the radius of the tube

3.4 Determination of surface tension by Jaegar's method

Principle:

The experiment is based on the principle that the pressure inside an air bubble liquid is greater than the pressure outside it by 2σ /r. Here a is the S.T. of the liquid and the radius of the air bubble. This excess pressure can be directly found and hence o can be calculated.

Apparatus:

An aspirator A is closed with a two- holed rubber stopper through which pass two glass tubes. One of these is connected to a water reservoir through a stopcock B and the other is joined through a tap C to a manometer M and a vertical tube DE. The tube DE ends in a narrow orifice at E and dips into the experimental liquid contained in a beaker.

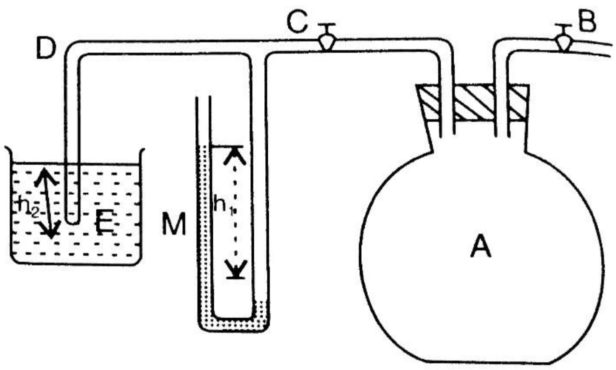


Figure 3. 3

Experimental details:

If the stopcock B is opened, water flows into the aspirator and the air in the aspirator is displaced. The displaced air forces its way through the tube DE and forms air bubbles at E. The size of each air bubble gradually grows. When its radius becomes equal to the radius of the tube at E, it becomes unstable and breaks away. During the growth of the bubble, the pressure inside increases and reaches a maximum value at the instant of detachment. The difference in manometer levels h_1 is noted just when the bubble detaches itself. At the moment of detachment,

The pressure inside the bubble $p_1 = H + h_1\, \rho_1\, g$

Where,

H = Atmospheric pressure,

h_1 = The difference n manometer levels

ρ_1 = Density of the manometric liquid

The pressure outside the bubble at the same time $p_2 = H + h_2\, \rho_2\, g$

Where,

h_2 = Length of the tube dipping in the experimental liquid

ρ_2 = Density of the manometric liquid

Excess pressure inside the bubble = 2σ /r

Hence, $\frac{2\sigma}{r} = (h_1\rho_1 - h_2\rho_2)g$ or $\sigma = \frac{1}{2} rg\,(h_1\rho_1 - h_2\rho_2)$

Advantages of the method:

- The angle of contact need not be known.
- The continual renewal of the liquid air interface helps in avoiding contamination.
- The experiment does not require a large quantity of liquid. (4) The liquid in the beaker may be heated to various temperatures. Hence the S.T. of a liquid can be determined at various temperatures.

Drawbacks:

- The exact value of the radius of the bubble when it breaks away cannot be ascertained.
- The drop may not be hemispherical and of quite the same radius as the aperture at E.
- The calculations are based on the assumption of static conditions, but the phenomenon is not entirely statical.
- For these reasons, this method does not give very accurate results for the surface tension.

3.5 Viscosity Introduction

Most fluids offer some resistance to motion, and we call this resistance "viscosity." Viscosity arises when there is relative motion between layers of the fluid. More precisely, it measures resistance to flow arising due to the internal friction between the fluid layers as they slip past one another when fluid flows. Viscosity can also be thought of as a measure of a fluid's thickness or its resistance to objects passing through it.

A fluid with large viscosity resists motion because its strong intermolecular forces give it a lot of internal friction, resisting the movement of layers past one another. On the contrary, a fluid with low viscosity flows easily because its molecular makeup results in very little friction when it is in motion. Gases also exhibit viscosity, but it is harder to notice in ordinary circumstances.

Definition:

Viscosity is a measure of a fluid's resistance to flow. The SI unit of viscosity is *poiseiulle (PI).* Its other units are Newton - second per square metre ($N\ s\ m^{-2}$) or pascal - second (*Pa s.)* The dimensional formula of viscosity is $[M\ L^{-1}\ T^{-1}]$.

The viscosity of liquids decreases rapidly with an increase in temperature, and the viscosity of gases increases with an increase in temperature. Thus, upon heating, liquids flow more easily, whereas gases flow more slowly. Also, viscosity does not change as the amount of matter changes, therefore it is an intensive property.

3.6 Rate of flow of liquid in a capillary tube

Consider horizontal capillary tube to length l and radius a through which a liquid flows is the coefficient of viscosity of the liquid. p is the pressure difference between the ends of the tube. The velocity of the liquid is maximum along and is zero at the walls. *(dv/dr)* is the velocity gradient.

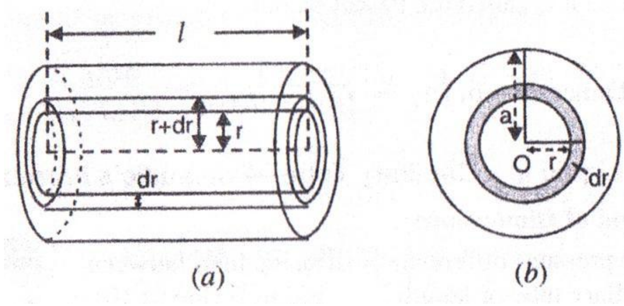
Consider a cylindrical shell of the liquid of inner radius r an outer radius *r+dr*

The surface area of the shell $A = 2\pi rl.$

According to Newton's law of viscous flow the backward dragging tangential force exerted by the outer layer on the inner layer is opposite to the direction of motion

$$F_1 = -\eta A \frac{dv}{dr}$$

$$= -\eta\, 2\, \pi\, r\, l \frac{dv}{dr}$$



The driving force on the liquid cell accelerating it forward

$$F_2 = p\, \pi\, r^2$$

Where, p = pressure difference across the two ends of the tube

$\pi\, r^2$ = Area of the cross section of the inner cylinder.

When the motion is steady,

Backward dragging force (F_1) = The driving force (F_2)

$$-\eta\, 2\, \pi\, r\, l\, \frac{dv}{dr} = p\, \pi\, r^2 \; or \; dv = \frac{-p}{2\eta\, l} r\, dr.$$

Integrating,

$$v = \frac{-p}{2\eta l}\frac{r^2}{2} + C$$

Where C is a constant of integration.

When r = a, v = 0. Hence,

$$0 = \frac{-p}{2\eta l}\frac{a^2}{2} + C$$

$$or \quad C = \frac{pa^2}{4\eta l}$$

$$v = \frac{\rho}{4\eta l}(a^2 - r^2)$$

This gives us the average velocity of the liquid flowing through the cylindrical shell. Hence the volume of the liquid that flows out per second through this shell

$$dV = \begin{pmatrix} Area\ of\ cross\ section\ of\ the\ shell \\ of\ radius\ r\ and\ thickness\ dr \end{pmatrix} \times Velocity\ of\ flow$$

$$2\,\pi r\ dr\ \frac{\rho}{4\eta l}(a^2 - r^2) = \frac{\pi p}{2\eta l}(a^2 r - r^3)$$

The volume of the liquid that flows out per second is obtained by integrating the expression for dV between the limits $r = 0$ and $r = a$.

$$V = \int_0^a \frac{\pi p}{2\eta l}(a^2 r - r^3)\ dr = \frac{\pi p}{2\eta l}\left[a^2 \frac{r^2}{2} - \frac{r^4}{4}\right]_0^a$$

$$= \frac{\pi p}{2\eta l}\ \frac{a^4}{4}$$

$$V = \frac{\pi p a^4}{8\eta l}$$

This above equation may give the rate of flow of a liquid through a capillary tube. This equation is also known as Posieuille’s formula.

3.7 Poiseuille’s formula and corrections

Two important corrections are to be applied in the Poiseuille's equation:

- Correction for pressure head
- Correction for length of tube

Correction for Pressure head

The outgoing liquid acquires K.E. due to its velocity after passing through the tube. Hence the pressure-head maintained is utilized not only for overcoming viscous resistance but also in imparting considerable K.E. to emergent liquid. So, the effective pressure is less and is given by

$$p_1 = p - \frac{V^2 \rho}{\pi^2 a^4}$$

This can be deducted as follows:

The K.E. given to the liquid of density ρ per second

$$E' = \int_0^a \frac{1}{2}(2\pi r\, drv\rho)v^2$$

$$= \pi\rho \int_0^a r\, v^3 dr$$

But $v = \frac{\rho}{4\eta l}(a^2 - r^2)$

$$E' = \pi\rho \int_0^a r\left(\frac{p}{4\,\eta\, l}\right)^3 (a^2 - r^2)^3 dr$$

$$= \pi\rho \left(\frac{p}{4\,\eta\, l}\right)^3 \frac{a^8}{8}$$

$$= \left(\frac{\pi\,\rho\, a^4}{8\,\eta\, l}\right)^3 \frac{\rho}{\pi^2 a^r}$$

$$= \frac{V^3\rho}{\pi^2 a^4}$$

The work done in overcoming viscosity is p_1V whereas total work done per unit volume is pV here p_1 is the effective pressure.

$$pV = p_1V + \frac{V^3\rho}{\pi^2 a^4}$$

$$p_1 = p - \frac{V^2\rho}{\pi^2 a^4}$$

$$p_1 = gp\left(h - \frac{V^2}{\pi^2 a^4 g}\right)$$

Thus, $\frac{V^2}{\pi^2 a^4 g}$ is the correction factor to the pressure head for gain of kinetic energy by the emergent liquid.

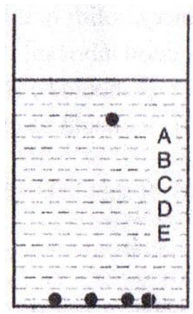
Correction for length of tube

At the inlet end of the tube, the flow of the liquid is not streamline for some distance. Consequently, the liquid is accelerated. The effective length of the tube is thus increased from *l* to *l +1.64a*. Thus, the corrected relation for η becomes,

$$\eta = \frac{\pi a^4}{8V\ (l + 1.64a)}\left(h - \frac{V^2}{\pi^2 a^4 g}\right)gp$$

3.8 Terminal velocity and Stoke's formula

Let us consider an infinite column of a highly viscous liquid like castor oil contained in a tall jar. If a steel ball is dropped into the liquid, it begins to move down with acceleration under gravitational pull. But its motion in the liquid is opposed by viscous forces in the liquid. These viscous forces increase as the velocity of the ball increases. Finally, a velocity will be attained when the apparent weight of the ball becomes equal to the retarding viscous forces acting on it. At this stage, the resultant force on the ball is zero. Therefore, the ball continues to move down with the same velocity thereafter. This uniform velocity is called the terminal velocity.



The viscous force *F* experienced by a falling sphere must depend on

- The terminal velocity *v* of the ball
- The radius *r* of the ball
- The coefficient of viscosity *η* of the liquid

We can write $F = k\, v^a\, r^b\, \eta^c$ where k is a dimensionless constant.

The dimensions of these quantities are $F = MLT^{-2}$; $v = LT^{-1}$; $r = L$; $\eta = ML^{-1}T^{-1}$

$$MLT^{-2} = (LT^{-1})^a\, (L)^b\, (ML^{-1}T^{-1})^c$$

$$MLT^{-2} = M^c\, L^{a+b-c}\, T^{-a-c}$$

Equating the powers of *M*, *L* and *T* on either side, $c = 1$; $a + b - c = 1$ and $c = -2$

Solving, $a = 1$; $b = 1$ and $c = 1$ Thus, $F = k\, v\, r\, \eta$

Stokes experimentally found the value of k to be 6π

$$F = 6\,\pi\, v\, r\, \eta$$

Expression for Terminal velocity

Leta ρ be the density of the ball and ρ' be the density of the liquid. Then,

The weight of the ball $= \frac{4}{3}\pi r^3 \rho g$

$$\left.\begin{matrix} \text{The weight of the displaced liquid} \\ \text{or the upthrust on the ball} \end{matrix}\right\} = \frac{4}{3}\pi r^3 \rho' g$$

$$\left.\begin{matrix} \text{The apparent weight} \\ \text{of the ball} \end{matrix}\right\} = \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \rho' g = \frac{4}{3}\pi r^3 (\rho - \rho') g$$

When the ball attains its terminal velocity *v*,

The apparent weight of the ball = viscous force *F*.

$$6\pi v r \eta = \frac{4}{3}\pi r^3 (\rho - \rho') g$$

$$v = \frac{2}{9}\frac{r^2}{\eta}(\rho - \rho') g$$

Assumptions made by Stokes while deriving the formula.

- The medium through which the body falls is infinite in extent.
- The moving body is perfectly rigid and smooth.
- There is no slip between the moving body and the medium.
- There are no eddy currents or waves set up in the medium due to the motion of the body through it. In other words, the body is moving very slowly through it.

3.9 Variation of viscosity with temperature

The viscosity of liquids decreases with temperature. For example, the viscosity of water decreases from 10^{-3} Nsm^2 at 20°C to 0.65 x 10^{-3} Nsm^2 at 40° C. However, there is no definite relation which expresses the variation of viscosity with temperature accurately. Slotte suggested the empirical formula

$$\eta_t = \frac{\eta_0}{1 + at + bt^2}$$

Where, η_t and η_0 are the viscosities of the liquid at t°C and 0°C respectively and a and b are constants. Andrade, on the basis theory of viscosity of liquids, derived the formula

$$\eta v^{\frac{1}{3}} = Ae^{c/vT}$$

Where A and c are constants, v is the specific volume of the liquid and T is the absolute temperature. This formula is in close agreement with the experimental results for all liquids so far examined except water and some tertiary alcohols.

Pressure produces a much smaller effect than temperature on the viscosity of liquids. Thus the viscosity of water decreases only marginally with increase of pressure up to a few hundred atmospheres.

The viscosity of ether at 20°C increases by only about 60% for an increase of 500 atmospheres of pressure. The change in viscosity with pressure is very much greater in the case of liquids with high viscosity. Barring the case of water whose viscosity decreases with pressure, the viscosity of all other liquids increases with pressure.

Unit 4: Waves and Oscillations

Simple Harmonic Motion (SHM) – differential equation of SHM – graphical representation of SHM – composition of two SHM in a straight line and at right angles –Lissajous's figures-free, damped, forced vibrations –resonance and Sharpness of resonance. Laws of transverse vibration in strings –sonometer – determination of AC frequency using sonometer – determination of frequency using Melde's string apparatus

4.1 Introduction

Oscillation is defined as the process of repeating variations of any quantity or measure about its equilibrium value in time. Oscillation can also be defined as a periodic variation of a matter between two values or about its central value.

The term vibration is used to describe the mechanical oscillations of an object. However, oscillations also occur in dynamic systems or, more accurately, in every field of science. Even the beating of our hearts creates oscillations. Objects that show motion around an equilibrium point are known as oscillators.

The most common examples of oscillation are the tides in the sea and the movement of a simple pendulum in a clock. Another example of oscillation is the movement of spring. The vibration of strings in guitars and other string instruments are also examples of oscillations.

Damped Oscillations

Damping is the process of restraining or controlling the oscillatory motion, such as mechanical vibrations, by the dissipation of energy. An oscillation remains undamped when a restoring force equal to the restraining for is induced, and hence the system oscillates with the same energy. When the restoring force is not applied, the oscillation suddenly stops. And when the restoring force applied is less than the restraining force, damping is introduced.

Damped oscillations are classified according to the difference in energy between the restoring force applied and the restraining force acting. A damped oscillation is an oscillation which fades away with respect to time, that is, the oscillations which reduce in magnitude with time.

4.2 Simple Harmonic Motion

Let *P* be a particle moving on the circumference of a circle of radius a with a uniform angular velocity ω. *O* is the centre of the circle.

A perpendicular *PM* is drawn from the particle on the diameter *YY'* of the circle. As the particle *P* moves round the circle, the foot of the perpendicular *M* vibrates along the diameter *YY'*. Since the motion of *P* is uniform, the motion of *M* is periodic. As the particle P completes one revolution, the foot of the perpendicular *M* completes one vertical oscillation. The distance *OM* is called the displacement and is denoted by y.

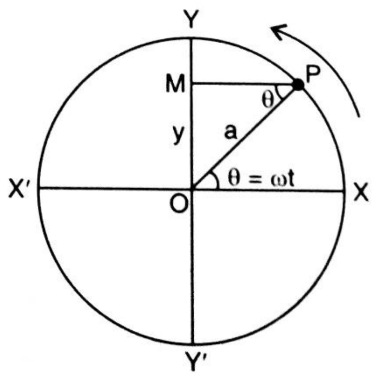


Figure 4. 1

The particle moves from *X* to *P* in time *t*.

$$\angle POX = \angle MPO = \theta = \omega\, t$$

From the ΔMPO,

$$\sin\theta = \sin\omega\, t = \frac{OM}{a}$$

$$OM = y = a \sin\omega\, t$$

OM is called the displacement of the vibrating particle. The displacement of a vibrating particle at any instant can be defined as its distance from the mean position of rest. The maximum displacement of a vibrating particle is called amplitude.

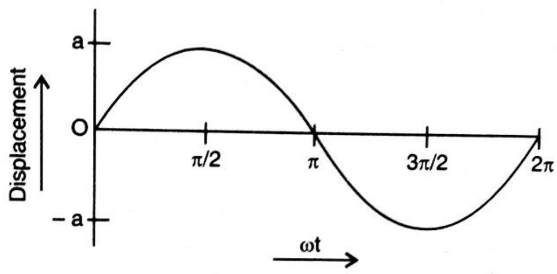


Figure 4. 2

Displacement = $y = a \sin\omega\, t$

The figure 4.2 shows the changes in the displacement of a vibrating particle in one complete vibration.

$$Velocity = v = \frac{dy}{dt} a\omega\, cos\omega t = \omega\sqrt{(a^2 - y^2)}$$

$$Acceleration = \frac{d^2y}{dt^2} - a\omega\, sin\omega t = \omega t - \omega^2 y \qquad (1),$$

acceleration is directly proportional to displacement and directed towards a fixed point. This type of motion is called Simple harmonic motion.

Characteristics

- The motion is periodic.
- The motion is along a straight line about the mean or equilibrium position.
- The acceleration is proportional to displacement.
- Acceleration is directed towards the mean or equilibrium position.

Definition

If a particle moves in a straight line, so that its acceleration is always directed towards a fixed point on the line and is proportional to its displacement from the fixed point, the particle is said to move with simple harmonic motion.

Equation (1) can be written as

$$\frac{d^2y}{dt^2} + \omega^2 y = 0$$

This is the differential equation of S.H.M

$$\text{The period } T = \frac{2\pi}{\omega}$$

$$\text{The frequency } n = \frac{1}{T}\frac{\omega}{2\pi}$$

Phase:

Consider a particle starting from S and moving on the circumference of a circle. $\angle SOX = \alpha$

This particle moves from S to P in time t.

$\angle OPM = (\omega t + \alpha)$

Displacement y = a sin (ω t + α)

Where,

α - Initial phase or epoch of the SHM

(ω t + α) - is called the phase of SHM

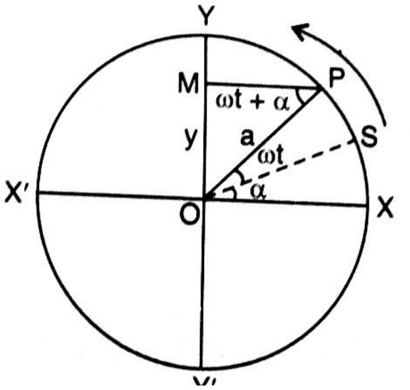


Figure 4. 3

Examples of SHM

- The vertical oscillations of a spiral spring suspended from a rigid support, and loaded at the lower end. This is linear type of SHM
- The vibrations of a simple pendulum. This is a angular type of SHM

4.3 Composition of two SHM in a straight line

When a particle is influenced by a number of collinear simple harmonic vibrations of different amplitudes and different epoch angles, the resultant vibration can be obtained by the polygon method. Iin this figure 4.4, *OP, PQ,* and *QR* are the vectors representing three simple harmonic vibrations. The amplitude of the individual vibrations are a_1, a_2 and a_3 and α_1, α_2 and α_3 are the corresponding epoch angles. The vector *OR* represents the resultant vibration. The amplitude and the epoch angle of the resultant vibration are *A* and ϕ respectively.

Proceeding in the same way this method can be employed when several collinear simple harmonic vibrations influence the same particle of the medium. Let n simple harmonic vibrations of the same amplitude a and epoch angles $0, 2\alpha, 4\alpha .. 2(n-1)\alpha$ influences a vibrating particle. If the displacements of the vibrating particle are considered along the y-axis, the individual displacements are given by,

$y_1 = a \sin(\omega t - 0)$

$y_2 = a \sin(\omega t - 2\alpha)$

Let *A* be the amplitude of the resultant vibration and ϕ the epoch angle. Then, the projections of the individual vectors *OP*, *PQ*, *QR* etc. on the y-axis are given by $0, a \sin 2\alpha, a \sin 4\alpha$, etc. Similarly, the projections on the X-axis are given by, $a, a \cos 2\alpha, a \cos 4\alpha$, etc.

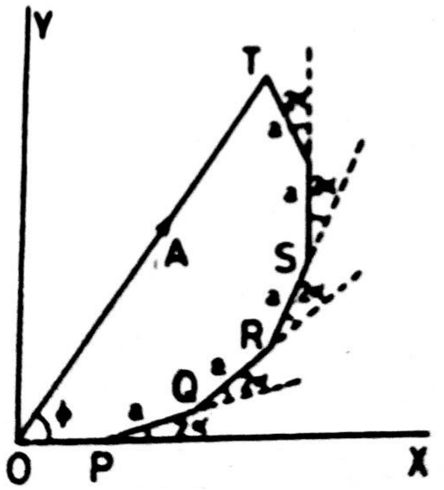


Figure 4. 4

If *OT* Represents the resultant vector, then $A \sin \phi$ will give the projection along Y-axis and A $\cos \phi$ gives the projection along the X-axis.

$$A \sin \phi = 0 + a \sin 2\alpha + a \sin 4\alpha + \ldots + a \sin 2(n-1)\alpha$$

$$= a[\sin 2\alpha + \sin 4\alpha + \ldots + \sin 2(n-1)\alpha] \quad (1)$$

Similarly,

$$A \cos \phi = a + a \cos 2\alpha + a \cos 4\alpha + \ldots. + a \sin 2(n-1)\alpha$$

$$= a[1 + \cos 2\alpha + \cos 4\alpha + \ldots. + \sin 2(n-1)\alpha] \quad (2)$$

Multiplying equation (2) by $2 \sin \alpha$

$$2A \cos \phi \sin \alpha = 2a \sin \alpha [1 + \cos 2\alpha + \cos 4\alpha + \ldots. + \sin 2(n-1)\alpha]$$

$$= a[2 \sin \alpha + 2 \cos 2\alpha . \sin \alpha + 2 \cos 4\alpha \sin \alpha + \ldots + 2 \cos 2(n-1)\alpha \sin \alpha]$$

$$= a[2 \sin \alpha + (\sin 3\alpha - \sin \alpha) + (\sin 5\alpha - \sin 3\alpha) + \ldots + \{\sin(2n-1)\alpha - \sin(2n - 3\alpha)\}]$$

$$= a[\sin \alpha + \sin(2n-1)\alpha]$$

$= a\,[2.\sin n\alpha \,.\, \cos (n-1)\,\alpha]$

Thus, $2A \cos \phi \ \sin \alpha = a\,[2.\sin n\alpha \,.\, \cos(n-1)\,\alpha]$

$$A \cos \phi = \frac{\alpha \sin\ n\alpha\,.\cos\,(n-1)\ \alpha]}{\sin \alpha} \qquad (3)$$

Multiplying equation (1) by 2 sin α and proceeding in a similar way, it can be shown that

$$A \sin \phi = \frac{\alpha \sin\ n\alpha\,.\sin\,(n-1)\ \alpha]}{\sin \alpha} \qquad (4)$$

Squaring equations (3) and (4) and adding,

$$A^2 (\sin^2 \phi + \cos^2 \phi) = A^2$$

$$= \frac{a^2 sin^2 n\alpha}{sin^2\alpha}\,[sin^2(n-1)\alpha + cos^2(n-1)\alpha]$$

$$= \frac{a^2 sin^2 n\alpha}{sin^2\alpha}$$

Thus,

$$A = \frac{a \sin n\alpha}{\sin \alpha}$$

Dividing equation (4) by (3)

$$\frac{A \sin \phi}{A \cos \phi} = \tan \phi = \frac{a \sin n\alpha\ .\sin(n-1)\alpha.\sin \alpha}{\sin \alpha\ .\ a \sin n\alpha\ .\cos(n-1)\alpha}$$

$$= \tan\,(n-1)\,\alpha$$

$$\Phi = (n-1)\,\alpha$$

Where ϕ represents the epoch angle for the resultant vibration, n is the number of simple harmonic vibrations influencing a particle and α is half the increase in the epoch angle between successive vibrations.

4.4 Composition of two SHM at right angles

Let

$$x = a \sin (\omega t + \alpha) \qquad \ldots (1)$$

$$y = b \sin \omega t \qquad \ldots (2)$$

Represent the displacements of a particle along the *X* and *Y* axes due to the influence of two simple harmonic vibrations acting simultaneously on a particle in perpendicular directions. Here, the two vibrations acting are of the same time period but are of different amplitudes and different phase angles. From equation (2),

$$\sin \omega t = \frac{y}{b}$$

$$\cos \omega t = \left[\sqrt{1 - \frac{y^2}{b^2}} \, . \sin \alpha \right]$$

From equation (1) $\frac{x}{a} = [\sin \omega t \cos \alpha + \cos \omega t \sin \alpha]$ … (3)

Substituting the values of sin ωt and cos ωt in equation (3)

$$\frac{x}{a} = \left[\frac{y}{b} \cos \alpha + \sqrt{1 - \frac{y^2}{b^2}} \, . \sin \alpha \right]$$

$$\frac{x}{a} - \frac{y}{b} \cos \alpha = \sqrt{1 - \frac{y^2}{b^2}} \, . \sin \alpha$$

Squaring,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} \, cos^2\alpha - \frac{2xy}{ab} \cos \alpha = \left(1 - \frac{y^2}{b^2} \right) sin^2\alpha$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} \, [sin^2\alpha + cos^2\alpha] - \frac{2xy}{ab} \cos \alpha = sin^2\alpha$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} \cos \alpha = sin^2\alpha$$

This represents the general equation of an ellipse. Thus, due to the superimposition of two. The displacement of the particle will be along a curve given by equation.

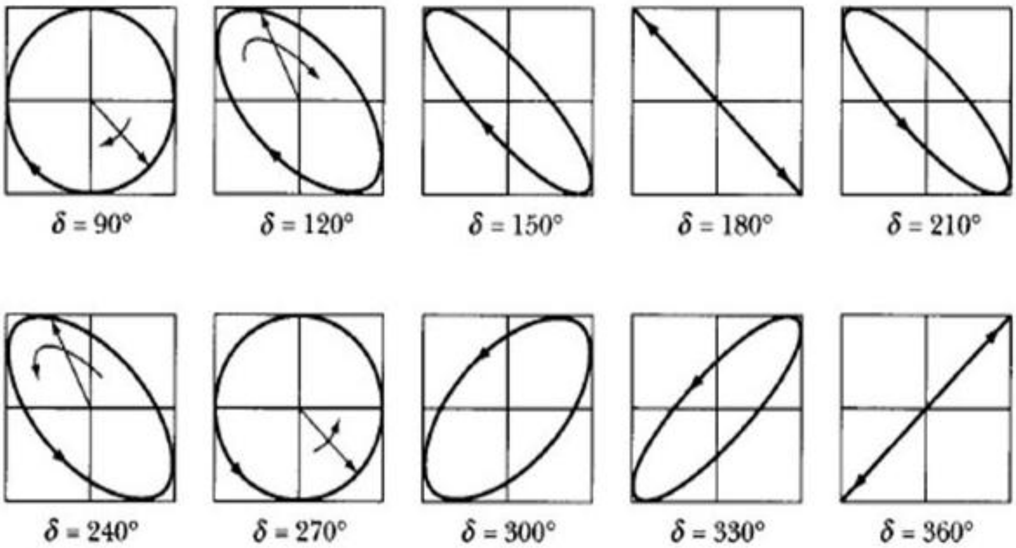


Figure 4. 5

The resultant vibration of the particle will depend upon the value of α. The above figure 4.5 represents the resultant vibration for values of α changing from 0 to 2π .

Special cases

i) If α = 0 or 2π; cos α = 1; sin α = 0

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{2xy}{ab}=0$$

$$\frac{x}{a}-\frac{y}{b}=0$$

$$y=\frac{b}{a}x$$

This represents the equation of the straight line BD i.e., the particle vibrates simple harmonically along the line DB.

ii) If α = π or 2π; cos α = -1; sin α = 0

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{2xy}{ab}=0$$

$$\frac{x}{a}+\frac{y}{b}=0$$

$$y=-\frac{b}{a}x$$

This represents the equation of the straight-line AC

iii) α = π / 2 or 3π/2 cos α = 0; sin α = 1

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+=1$$

This represents the equation of the ellipse with a and b as the semi-major and semi-minor axes.

iv) α = π / 2 or 3π/2; a = b;

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$x^2+y^2=a^2$$

This represents the equation of a circle

4.5 Lissajous's figures-free, damped, forced vibrations.

Free Vibrations

When the bob of a simple pendulum (in vacuum) is displaced from its mean position and left, it executes simple harmonic motion. The time period of oscillation depends only on the length of the pendulum and the acceleration due to gravity at the place. The pendulum will continue to oscillate with the same time period and amplitude for any length of time. In such cases there is no loss of energy by friction or otherwise. In all similar cases, the vibrations will be undamped free vibrations. The amplitude of swing remains constant.

Damped Vibrations

In actual practice, when the pendulum vibrates in air medium there are frictional forces and consequently energy s dissipated in each vibration. The amplitude of swing decreases continuously with time and finally the oscillations die out. Such vibrations are called free damped vibrations. The dissipated energy appears as heat either within the system itself or in the surrounding medium. The dissipative force due to friction etc. (resistance in LCR Circuit) is proportional to the velocity of the particle at that instant. Let $\mu \frac{dy}{dt}$ be the dissipative force due to friction etc.

Therefore, the differential equation in the case of free damped vibrations is

$$m\frac{d^2y}{dt^2} + Ky + \mu\left(\frac{dy}{dt}\right) = 0 \qquad (1)$$

$$\frac{d^2y}{dt^2} + \left(\frac{\mu}{m}\right)\frac{dy}{dt} + \left(\frac{K}{m}\right)y = 0 \qquad (2)$$

This equation is similar to a general differential equation,

$$\frac{d^2y}{dt^2} + 2b\frac{dy}{dt} + k^2y = 0 \qquad (3)$$

The solution of this equation is

$$y = ae^{-bt}\sin(\omega t - \alpha) \qquad (4)$$

The general solution of equation (2) is also given by

$$y = Ae^{\left(-b+\sqrt{b^2-k^2}\right)}t + \; -Be^{\left(-b+\sqrt{b^2-k^2}\right)}t$$

Here,

$$b = \frac{\mu}{2m} \quad and \quad k^2 = \frac{K}{m}$$

$$\omega = \sqrt{k^2 - b^2}$$

$$\omega = \sqrt{\frac{K}{m} - \frac{\mu^2}{4m^2}}$$

$$n = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{k^2 - b^2}$$

Forced Vibration

The time period of a body executing simple harmonic motion depends on the dimensions of the body and its elastic properties. The vibrations of such a body die out with time due to dissipation of energy. If some external periodic force is constantly applied on the body, the body continues to oscillate under the influence of such external forces. Such vibrations of the body are called forced vibrations. Initially, the amplitude of the swing increases, then decreases with time, becomes minimum and again increases. This will be repeated if the external periodic force is constantly applied on the system. In such cases the body will finally be forced to vibrate with the same frequency as that of the applied force. The frequency of the forced vibration is different from the natural frequency of vibration of the body. The amplitude of the forced vibration of the body depends on the difference between the natural frequency and the frequency of the applied force. The amplitude will be large if difference in frequencies is small and vice versa.

For forced vibrations, equation (1) is modified in the form

$$m\frac{d^2y}{dt^2} + Ky + \mu\left(\frac{dy}{dt}\right) = F\sin pt \qquad (5)$$

Here p is the angular frequency of the applied periodic force.

The particular solution of equation (5) representing the forced vibration is

$$y = a\sin(pt - \alpha)$$

$$\frac{dy}{dt} = ap\cos(pt - \alpha)$$

$$\frac{d^2y}{dt^2} = -ap^2\sin(pt - \alpha) = -p^2y$$

Substituting these values in equation (5)

$- mp^2 a \sin(pt - \alpha) + Ka \sin(pt - \alpha) + \mu a p \cos(pt - \alpha) = F \sin pt$

$- mp^2 a [\sin pt \cos\alpha - \cos pt \sin\alpha] + Ka [\sin pt \cos\alpha - \cos pt \sin\alpha]$

$+ \mu a p [\cos pt \cos\alpha + \sin pt \sin\alpha] - F \sin pt = 0$

When *sin pt =1; cos pt = 0*

$- \; mp^2 a \cos\alpha + Ka \cos\alpha + \mu a p \sin\alpha - F = 0$ (6)

When *cos pt = 1; sin pt = 0*

$mp^2 a \sin\alpha - Ka \sin\alpha + \mu ap \sin\alpha - F = 0$ (7)

Dividing (7) by *cos α* and simplifying

$$\tan\alpha = \frac{\mu p}{K - mp^2} = \frac{A}{B}$$

$$\sin\alpha = \frac{A}{\sqrt{A^2 + B^2}}$$

$$\cos\alpha = \frac{B}{\sqrt{A^2 + B^2}}$$

Dividing equation (6) by cos α

$$-\mathrm{m}p^2\mathrm{a} + \mathrm{Ka} + \mu\,\mathrm{a\,p}\tan\alpha = \frac{F}{\cos\alpha}$$

$$\mathrm{a}\,[(\mathrm{K} - \mathrm{m}p^2) + \mu\mathrm{p}\tan\alpha] = \frac{F}{\cos\alpha}$$

$$\mathrm{K} - \mathrm{m}p^2 = \mathrm{B}, \text{and } \mu\mathrm{p} = \mathrm{A}$$

Substituting the values of *A* and *B*

$$a = \frac{F}{\sqrt{\mu^2 p^2 + (K - mp^2)^2}}$$

$$y = a\sin(pt - \alpha)$$

$$y = \frac{F}{\sqrt{\mu^2 p^2 + (K - mp^2)^2}}\sin(pt - \alpha)$$

Applying the boundary conditions, another solution is obtained when *F =0*. This corresponds to free vibrations. In the case of free vibrations, the solution is

$$y = ae^{-bt}\sin(\omega t - \alpha)$$

The general solution will include both the particular solutions for free and forced vibrations.

$$y = e^{-bt}\sin(\omega t - \alpha) + \frac{F}{\sqrt{\mu^2 p^2 + (K - mp^2)^2}}\sin(pt - \alpha)$$

$$b = \frac{\mu}{2m}$$

4.6 Resonance and Sharpness of resonance

In the case of forced vibrations, the general solution for the displacement at any instant is given by

$$y = e^{-bt}\sin(\omega t-\alpha)+\frac{F}{\sqrt{\mu^2p^2+(K-mp^2)^2}}\sin(pt-\alpha)$$

If the effect of viscosity of the medium is small, the amplitude,

$$\frac{F}{\sqrt{\mu^2p^2+(K-mp^2)^2}}$$

Under the action of the driving force is maximum when the denominator is minimum. This is possible if $K - mp^2 = 0$ *or* $K = mp^2$

$$p = \sqrt{\frac{K}{m}}$$

Further, the amplitude will be infinite if μ also zero. The oscillations will have maximum amplitude and this state of vibrations of a system is called resonance. IT means that, when the forced frequency is equal to the natural frequency of vibrations of the body, resonance takes place. If friction is present, the amplitude at resonance

$$=\frac{F}{\mu p}=\frac{F}{\mu\sqrt{\frac{K}{m}}}$$

Or amplitude at resonance

$$=\frac{F}{\mu}\sqrt{\frac{m}{K}}$$

In the case of sound, the study of sharpness of resonance is of great importance. Sharpness of resonance refers to the fall in amplitude with change in frequency on each side of the maximum amplitude.

The particular solution for displacement in the case of forced vibrations is,

$$y=\frac{F}{\sqrt{\mu^2p^2+(K-mp^2)^2}}\sin(pt-\alpha) \qquad (1)$$

Differentiating equation (1) with respect to time

$$\frac{dy}{dt}=\frac{FP}{\sqrt{\mu^2p^2+(K-mp^2)^2}}\sin(pt-\alpha)$$

The velocity (dy/dt) is maximum when cos (pt – α) is maximum i.e., the instant at which the particle crosses the mean position.

$$\left(\frac{dy}{dt}\right)_{max} = \frac{FP}{\sqrt{\mu^2p^2+(K-mp^2)^2}}$$

Kinetic energy of the vibrating particle at the instant of crossing the mean position is given by

$$K.E = \frac{1}{2}m\left(\frac{dy}{dt}\right)_{max}^2$$

$$K.E = \frac{\frac{1}{2}mF^2p^2}{\mu^2p^2+(k-mp^2)^2} \qquad (2)$$

The mean square of the driving force per unit mass

$$= \frac{\frac{O+F^2}{2}}{m} = \frac{F^2}{2m}$$

Dividing equation (2) by $F^2/2m$ we get kinetic energy per unit force which is called the response R.

$$R = \frac{\frac{1}{2}mF^2p^2}{\mu^2p^2+(k-mp^2)^2} \div \frac{F^2}{2m}$$

$$R = \frac{m^2p^2}{\mu^2p^2+(k-mp^2)^2}$$

$$R = \frac{p^2}{\frac{\mu^2p^2}{m^2}+\left(\frac{k}{m}-p^2\right)^2} \qquad (3)$$

The natural frequency of the system in the absence of damping is $\sqrt{\frac{K}{m}}$ Therefore, the term $\left(\frac{K}{m}-p^2\right)$ *in equation* (3) represents the extent to which the natural frequency of the system deviates from the forced frequency.

When $\frac{K}{m} = p^2$

The natural frequency coincides with the forced frequency and the value of R will be maximum. From equation (3)

$$R = \frac{p^2}{\frac{\mu^2p^2}{m^2}} = \frac{m^2}{\mu^2} = \left(\frac{m}{\mu}\right)^2$$

$$R \propto \frac{1}{\mu}$$

It means that the response R is inversely proportional to the frictional force. IN the absence of friction, the response is maximum. The term $\left(\frac{K}{m} - p^2\right)$in equation (3), refers to mistuning. The larger is its value, the greater is the system away from resonance. The graph between *p/ω* along the X-axis and the response *R* along the Y-axis is shown below.

- When p/ω is equal to 1 the response is maximum. For curve *A*, μ is large and curve *C*, μ is less. The response decreases for values p/ω greater than 1 or less than 1.
- When the frictional forces are absent, i.e., μ=0, R is infinite and the sharpness of resonance is maximum.

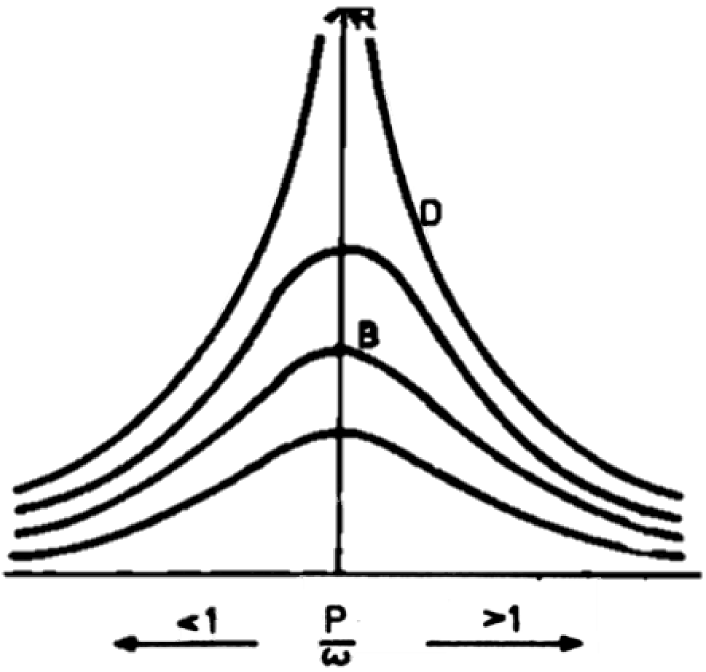


Figure 4. 6

- The sharpness of resonance decreases with increase in the value of μ.
- The sharpness of resonance dies rapidly even for a very small change in the value of *p/ω* from 1, in this case, where μ is minimum.

In the case of the resonance tube, the damping force is large and the graph will be similar to the curve A in the graph. The resonance persists over a wide range and it is difficult to exactly locate the position of maximum sharpness of resonance. Hence the results obtained with the resonance tube apparatus are not very accurate.

In the case of the sonometer wire, the damping forces are small and the graph will be similar to curve C in the graph. In this case, the sharpness of resonance is maximum in a very narrow region. Even a slight variation in length or tension reduces the sharpness considerably. The vibrations die out rapidly. Thus, the results obtained with a sonometer are accurate.

4.7 Laws of transverse vibration in strings

Velocity of a transverse wave travelling in stretched string is given by:

$$v = \sqrt{\frac{T}{\mu}}$$

Where T is the tension in the string and μ is the mass per unit length.

Now, the fundamental frequency of the stretched string is given by:

$$f = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

l is the resonating length.

From the above expression, there are three laws of transverse vibration of string.

Law of length: The fundamental frequency is directly proportional to the resonating length (L) of the string.

From the above expression, there are three laws of transverse vibration of string.

Law of length:

The fundamental frequency is directly proportional to the resonating length (L) of the string.

$$f \propto \frac{1}{L}$$

Verification:

When we take different tuning forks of different frequencies, and measure the resonating length for each of them keeping tension applied and the material of the wire as constant. The product of frequency and resonating length of one tuning fork was found equal to the frequency and resonating length of another tuning fork. This verifies the law of length.

Law of Tension:

The fundamental frequency is directly proportional to the square root of the tension.

$$f \propto \sqrt{T}$$

Verification:

The resonance of different tuning forks of different frequencies was observed by varying tension keeping resonating length and wire as constant. When a graph for *f* vs √T is plotted, a straight line is obtained. This verifies the law of tension.

Law of Mass:

The fundamental frequency is inversely proportional to the square root of the mass per unit length.

$$f \propto \frac{1}{\mu}$$

Verification:

The resonance is observed by taking different tuning forks of different frequencies and different wires with separate mass per unit length (μ) keeping resonating length and tension applied as constant. When the graph for *f* vs $1\sqrt{\mu}$ is plotted and the graph is found to be a straight line. So, law of mass is verified.

4.8 Determination of AC frequency using sonometer

Apparatus Required

A sonometer with soft wires, a set of eight tuning forks, seven ½ kg slotted weights, A set of eight tuning forks, Clamp, Rubber Pad, Paper rider, Meter Scale

Theory

Let us consider the alternating current to have a frequency v so that the frequency of magnetization of the electromagnet VE becomes 2v.

Let a loaded stretched soft iron wire have a resonant length l_1 with the electromagnet. Let a tuning fork of frequency V_t have resonant length l_2

$$v_E l_1 = V_r l_2$$

$$v_E = V_r \frac{l_2}{l_1}$$

Hence, the frequency of the alternating current is calculated using the formula

$$n = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

$$\mu = \frac{mass}{length} = \frac{m}{l}$$

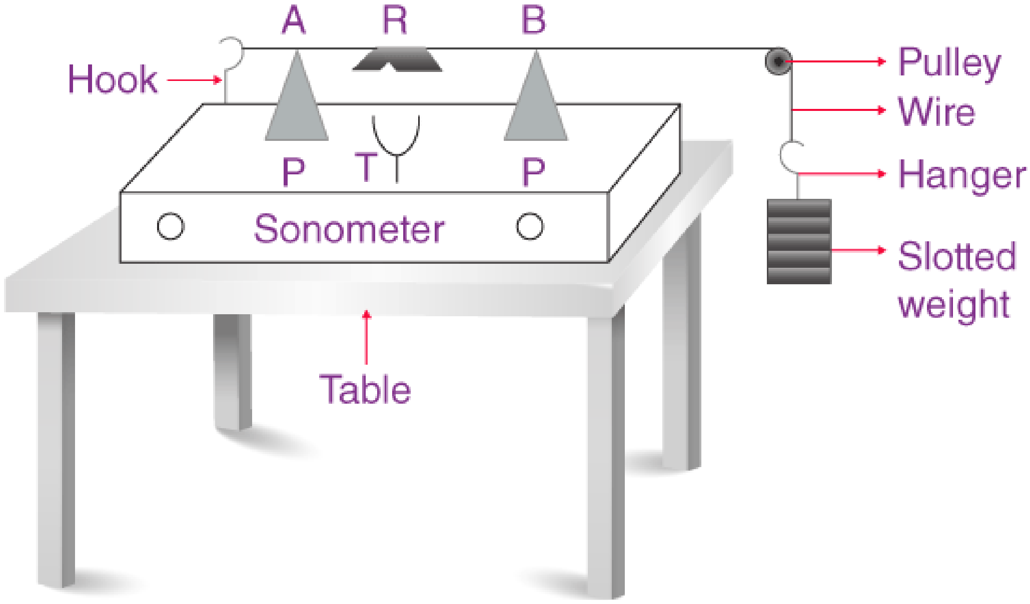


Figure 4. 7

Procedure

- Place the sonometer on the table as shown in the figure 4.8.
- Test the pulley and make it frictionless by oiling.
- Put suitable weights in the hanger.
- Move the wooden bridge P outward to include the maximum length of wire AB.
- Decrease the length of the wire by moving the wooden bridges equally inwardly.
- Go on decreasing the length till the sonometer wire starts vibrating.
- The length of the wire can be adjusted for the maximum amplitude of vibration.
- Measure the length of the wire AB between the edges of the two bridges and record it in length decreasing columns.
- Bring the bridges closer and adjust the length for the maximum amplitude by increasing it.
- Measure the length and record it in length increasing column.
- Now take a tuning fork of minimum known frequency and adjust the wire length with the vibrating tuning fork.

- Repeat step 11 above with tuning forks of other known frequencies.
- Record your observations.

Calculation

- Using the formula, $vE = v_2\, l_2 / l_1$, calculate v_E with observations 2 to 7.
- Record these values in column 4 of the table.
- Find the mean of the above six values of v_E.
- Then the frequency of the alternating current can be determined by the formula,
  $v = v_E /2$

$$n = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

for each observation and take the mean. Lastly, compare it with the standard frequency (50 Hz)

4.9 Determination of frequency using Melde's string apparatus

In Melde's Experiment, one end of the string is to the prong of electrically maintained tuning fork. The other end is connected to the scale pan. The string passes over a smooth frictionless pulley. The distance between the tuning fork and the pulley can be adjusted. There are two modes of vibration

(i) transverse mode of vibration

(ii) longitudinal mode of vibration

Transverse mode of vibration

The tuning fork vibrates at right angles to the length of the string. In this case the frequency of vibration of the string is equal to the frequency of the tuning fork. Suppose N is the frequency of the tuning fork and the string of length l vibrates in p_1 segments.

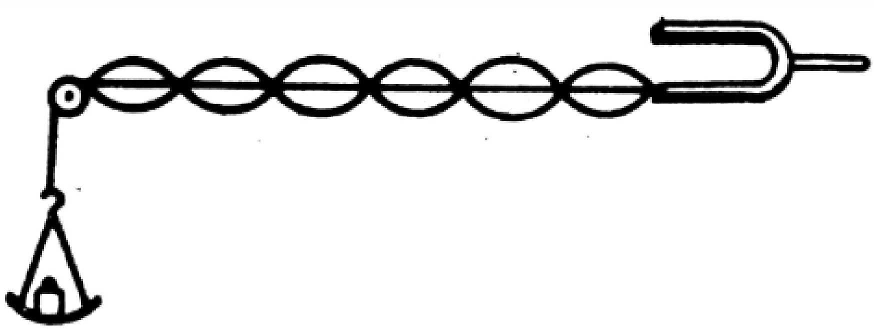


Figure 4. 8

$$N = \frac{p_1}{2l}\sqrt{\frac{T}{m}} \qquad (1)$$

Longitudinal mode of vibration

The tuning fork vibrates along the direction of the length of the string. In this case for one complete vibration of the tuning fork the string completes half vibration. Suppose the frequency of the tuning fork is *N*. therefore, the frequency of vibration of the string is *N/2*. If the string of length *l* vibrates in p_2 segments, the frequency of vibration of the string,

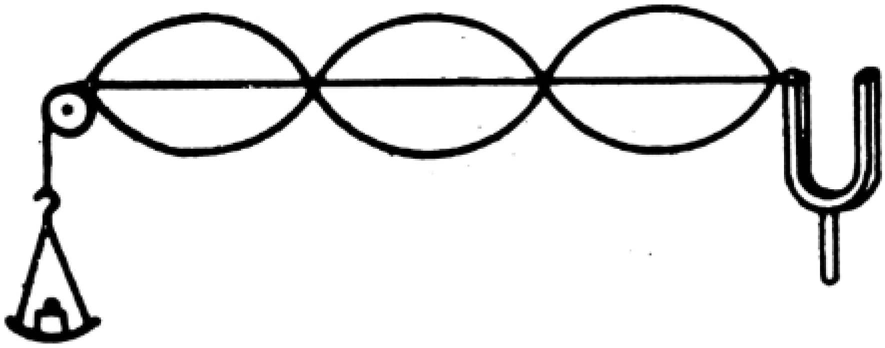


Figure 4. 9

$$\frac{N}{2} = \frac{p_2}{2l}\sqrt{\frac{T}{m}} \qquad (2)$$

Suppose the string vibrates in p segments in the transverse mode, then for the same tuning fork and the same tension, the string will vibrate in half the number of segments in longitudinal mode of vibration.

From equation (1) and (2),

$$p_2 = \frac{p_1}{2}$$

From equation (1), for the transverse mode of vibration

$$N = \frac{p_1}{2l}\sqrt{\frac{T}{m}}$$

$$N^2 = \frac{p_1^2}{4l^2 m}$$

$$Tp_1^2 = 4N^2 l^2 m = constant$$

$$Tp^2 = constant$$

Unit 5: Acoustics of Buildings and Ultrasonics

Intensity of sound – decibel – loudness of sound –reverberation – Sabine's reverberation formula – acoustic intensity – factors affecting the acoustics of buildings.

Ultrasonic waves: production of ultrasonic waves – Piezoelectric crystal method – magneto striction effect – application of ultrasonic waves

5.1 Introduction

The branch of physics that is concerned with the study of sound is known as acoustics. We can define acoustics as, the science that deals with the study of sound and its production, transmission, and effects.

A scientist or researcher who studies acoustics is called an Acoustician and someone working in the field of acoustics technology will be called Acoustical Engineer. The main application of acoustics is to make the music or speech sound as good as possible. It is achieved by reducing the sound barriers and increasing the factors that help in the proper transmission of sound waves. Initially, acoustics was used only in industries which are based on sound like an auditorium, or theatre but today, the application of acoustics has spread to many fields.

Acoustic Energy

Acoustic energy can be defined as the disturbance of energy which passes through matter in the form of a wave. In other words, it is the energy concerning the mechanical vibrations from its components is called acoustic energy. Any acoustic event has the following stages.

- Cause or Generating Mechanism
- Acoustic wave propagation
- Reception Effect

The process in which some other form of energy is converted into sonic energy producing a sound wave is called the transduction process. Sound waves carry energy throughout the propagating medium. The acoustic wave equation is the fundamental equation that describes sound wave propagation. Wave propagation is the key process in any acoustic event. Sound propagates in liquids as a pressure wave and in solids as longitudinal or transverse waves.

Environmental Noise

Environmental acoustics is concerned with vibration and noise caused by roadways. Railways, aircraft and general activities that are related to the environment. The main goal of these is to reduce vibration and noise that affects the environment.

Musical Acoustics

Musical acoustics is concerned with the study of physics of music i.e., how sounds are used to make music. Areas of study include human voice, musical instruments, and music therapy.

Ultrasonics

Ultrasonics or Ultrasounds are the sounds with a frequency greater than the human audible limit. However, there is no difference in physical properties when compared to normal sound. Ultrasound is used in many fields. Ultrasonic devices are used in measuring distances and in detecting objects. Ultrasound imaging is used in physics.

Infrasonics

Infrasonics are Infrasounds are the sounds with a frequency of less than 20 Hz. The study of such sounds is called infrasonics. Applications include detection of petrol formation under the earth and the possibility of earthquakes.

Vibration and Dynamics

It is the study of how mechanical systems vibrate and interact with their environment. Applications include vibration control which helps to protect a building from earthquakes and ground vibrations used in railways.

5.2 Intensity of sound – Decibel

The intensity of sound is defined as the average rate of transfer energy per unit area, the area being perpendicular to the direction of propagation of sound. Determination of the intensity of sound is important in practical acoustics.

Amount of energy transfer per unit area per second

$$I = 2\pi^2 \rho n^2 a^2 v \qquad (1)$$

$$v = \sqrt{\frac{E}{\rho}} \qquad and \qquad E = -\frac{P}{dV/V}$$

dV is the change in volume, *V* the original volume and *p* is the excess of pressure

$$p = \sqrt{\frac{-p}{\left(\frac{dV}{V}\right)p}}$$

$$Taking \qquad \frac{dV}{V} = \frac{dy}{dx} \qquad and\ simplifing$$

$$p = -v^2\rho\frac{dy}{dx} \tag{2}$$

A simple harmonic wave is represented by the equation

$$y = a\sin\frac{2\pi}{\lambda}(vt - x)$$

$$\frac{dy}{dx} = -\frac{2\pi a}{\lambda}\cos\frac{2\pi}{\lambda}(vt - x)$$

Substituting this value of *dy/dx* in equation (2)

$$p = \frac{2\pi a v^2 p}{\lambda}\cos\frac{2\pi}{\lambda}(vt - x)$$

The maxim excess of pressure

$$p_{max} = \frac{2\pi a v^2 \rho}{\lambda}$$

$$p = p_{max}\cos\frac{2\pi}{\lambda}(vt - x)$$

$$p_{max} = 2\pi a p v\frac{v}{\lambda}$$

$$p_{max} = 2\pi a p v n \tag{3}$$

From equation (1) and (6)

$$I = 2\pi^2\rho n^2 a^2 v$$

$$I = \frac{(2\pi\rho v n)^2}{2pv}$$

$$I = \frac{p_{max}^2}{2pv} \tag{4}$$

Equation (4) shows that the intensity of sound varies directly as the square of the excess of pressure. Therefore, in acoustics it is important to measure the excess of pressure to obtain the value of intensity of sound. For ordinary conversation p_{max} = 0.1 newton/m^2.

In the case of ordinary conversation, the sound output per square meter is 1.13×10^{-5}watt. Human ear is an extremely sensitive organ and can detect sound intensity lower than this value. It has been found that the ear can detect intensities as low as 10^{-12} watt/m^2

5.3 Measurement of sound - Decibel

The intensity of sound is defined as the quantity of energy propagating through a unit area per unit time, the direction of propagation being perpendicular to the area. The unit of intensity in the CGS system is *ergs/cm^2-s.* and in SI system it is *joules/m^2-s.* The amount of power transmitted per unit area is measured in watts/m^2. A convenient unit is micro-watts/m^2.

The loudness of sound is just an aural sensation and it is a physiological phenomenon rather than a physical phenomenon. The intensity of sound refers to the external or the objective measurement and the loudness refers to the internal or subjective aspect. Intensity of sound is a definite physical quantity and loudness is merely a degree of sensation. Loudness of sound increases with the intensity of sound according to Weber - fecher law in physiology. According to this law does not hold good near the lower and the upper limits of audibility. According to this law,

$$S \propto \log I$$

$$S = K \log I$$

Where K is a constant

$$\frac{dS}{dI} = \frac{K}{I}$$

The quantity *dS/dI* is called the sensitiveness of the ear. The sensitiveness of the ear decreases with increase in the intensity of sound.

In the case of all practical measurements, it is the relative intensity that is important and not the absolute value. Hence the intensity of sound is often measured as its ratio to a standard intensity I_0. The intensity level is equal to I/I_0

The standard intensity taken as 0*.01 watt/m^2*

Suppose a person speaks in a normal conversational tone, he emits energy at the rate 10^{-5} joule/s. The mouth aperture is about 10^{-3} m^2 while speaking. If he of the tube a short tube the whole of the sound energy spreads along the tube and the intensity of sound is 10^{-2} *watt/ m^2*. This value of sound intensity is the standard intensity I_0. The person hearing at the other end of the tube gets the feeling of standard intensity. If a person shouts into the tube as loud as he can, the intensity will be 100 I_0. When the intensity is 100 I_0 up to 1000 I the listener feels pain.

The faintest sound that can be heard depends also upon the frequency of the note. The average person's threshold of audibility is about 10^{-10} I_0 for a frequency of 400 hertz. Thus the range of hearing for a human ear is from 10^{-10} up to 100 I_0. Hence the human ear has a dynamic range of 10^{12} in intensity.

5.4 Loudness of sound

The amount of sound energy crossing per unit area around a point in one second is known as intensity of sound. Loudness depends upon intensity and also upon the sensitiveness of the ear. Loudness and intensity are related to each other by the relation

$$L \propto \log I$$

where L represents the sensations of loudness and 1, the intensity of sound.
Loudness or intensity depends upon the following factors:

(i) Amplitude

Loudness is directly proportional to the square of the amplitude of the sounding body. The amplitude of sound produced by men is large and hence loud sound is produced. The amplitude of sound produced by ladies or children is small, therefore, the sound produced is feeble. Mosquito also produces a wave of small amplitude; therefore, the sound produced by a mosquito is also feeble.

(ii) Surface area

Loudness is directly proportional to the surface area of the sounding body. A tuning fork of large size produces a loud sound as compared to a tuning fork of small size. Beating drums with large surfaces produce a loud sound as compared to the beating drums with small surface area. A tuning fork ordinarily produces a feeble sound. When its stem is pressed against a table, a loud sound is produced. The particles of the table are forced to vibrate with the frequency of the tuning fork and the apparent surface area increases, hence a loud sound is produced.

(iii) Distance between the source and the listener.

The intensity of sound is inversely proportional to the square of the distance between the source and the listener, provided the source produces sound waves in all directions. There- fore, the sound becomes feeble and feeble with increase in distance between the listener and the source.

(iv) Density of the medium

The greater the density of the medium, the louder is the sound. When the density of the medium is decreased, the sound becomes feeble.

(v) Motion of air

If air is blowing in the direction of propagation of the sound waves, loudness increases. If air is blowing in a direction opposite to the direction of propagation of the sound waves, loudness decreases

5.5 Reverberation

It is observed that for a listener in a room or an auditorium, whenever a sound pulse is produced, he receives directly compressional sound whenever a source, as well as sound waves from the walls, ceiling and other materials present in the room. The waves received by the listener are:

(i) direct waves

(ii) reflected waves

due to multiple reflections at the various surfaces. The quality of the note received by the listener will be the combined effect of these two sets of waves. There is also a time gap between the direct wave received by the listener and the waves received by successive reflection. Due to this, the sound persists for some time even after the source has stopped. This persistence of sound is termed as reverberation. The time gap between the initial direct note and the reflected note up to the minimum audibility level is called reverberation time. The reverberation time will depend on the size of the room or the auditorium, the nature of the reflecting material on the wall and the ceiling and the area of the reflecting surfaces.

In a good auditorium it is necessary to keep the reverberation time negligibly small. The intensity of sound as received by the listener, is shown graphically in Figure 5.1. When a source emits sound, the waves spread out and the listener is aware of the commencement of sound when the direct waves reach his ears. Subsequently the listener receives sound energy due to reflected waves also. If the note is continuously sounded, the intensity of sound at the listener's ears gradually increases. After sometime a balance is reached between the energy emitted per second by the source and the energy lost or dissipated by walls or other materials. The resultant energy attains an average steady value, and to the listener the intensity of sound appears to be steady and constant. This is represented by the portion BC of the curve ABCD. If at C, the source stops emitting sound, the intensity of sound falls exponentially as shown by the curve CD. When the intensity of sound falls below the minimum audibility level, the listener will not hear the sound. When a series of notes are produced in an auditorium (say speech or music)v each note will give rise to its own intensity curve with respect to time. The curves for these notes are shown in Fig.

For clear audibility of speech or music, it is necessary that

(i) each separate note should give sufficient intensity of sound in every part of the auditorium, and

(ii) each note should die down rapidly before the maximum average intensity due to the next note is heard by the listener. This is particularly important with speech. In the case of music comparatively more reverberation can be tolerated.

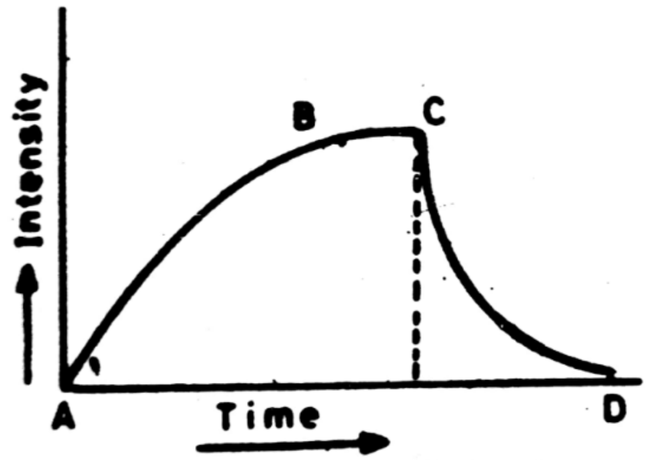


Figure 5. 2

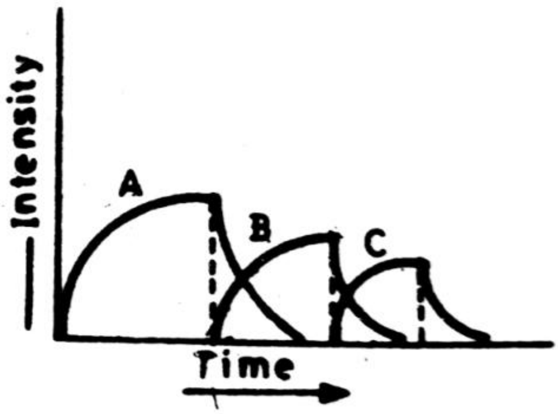


Figure 5. 1

5.6 Sabine’s reverberation formula

Sabine developed the reverberation formula to express the rise and fall of sound in an auditorium. The main assumptions are:

(1) The average energy per unit volume is uniform. It is represented as o.

(2) The energy is not lost in the auditorium. The energy lost is only due to the absorption of the material of the walls and ceiling and also due to the escape through the windows and ventilators. Both these factors are included in the term 'absorption' of energy.

Suppose a source is producing sound continuously. This sound energy is propagated in all directions. Let σ be the energy contained in a unit volume. The energy that contained in a solid angle *dφ*

$$= \frac{\sigma . d\varphi}{4\pi}$$

Let this energy be incident on a unit surface area of the wall at an angle θ.

If the velocity of sound is *v*,

then the total energy falling per second on a unit surface area of the wall.

$$= \left(\frac{\sigma . d\varphi}{4\pi}\right) \cos\theta \ . v$$

The total energy falling per second within a hemisphere

$$= \frac{\sigma v}{4\pi} . \int \cos\theta . d\varphi$$

$$\varphi = 2\pi \ (1 - \cos\theta)$$

$$d\varphi = 2\pi \sin\theta . d\theta$$

Substituting this value of *dφ* ,

The total energy falling per second within a hemisphere

$$= \frac{\sigma v}{4\pi} \int_0^{\pi/2} 2\pi \sin\theta . \cos\theta . d\theta$$

$$= \frac{\sigma v}{2} \left[- \frac{cos^2\theta}{2} \right]_0^{\frac{\pi}{2}}$$

$$= \frac{\sigma v}{4}$$

Suppose α is the absorption coefficient of the walls that refers to the fraction of the incident energy not reflected from the walls. The amount of energy absorbed per second unit area = $\frac{\alpha\sigma v}{4}$. If A is the area of the walls and the amount of energy absorbed per second = $\frac{A\alpha\sigma v}{4}$.

Let V be the volume of the auditorium, the total energy = Vσ.

The rate of increase of energy = $\frac{d}{dt} V\sigma \quad = v \frac{d\sigma}{dt}$ (1)

Suppose, the source supplies energy at the rate of Q units per second.

Then the rate of increase of energy = $Q - \frac{A\alpha\sigma v}{4}$ (2)

Equating (1) and (2)

$$v\frac{d\sigma}{dt} = \ Q - \frac{A\alpha\sigma v}{4}$$

$$\frac{A\alpha\sigma v}{4} = K \qquad (3)$$

$$\frac{K}{V} = \beta \quad and\ B = \frac{Q}{K} = \frac{4Q}{A\alpha v}$$

From equation (3),

$$V\frac{d\sigma}{dt} = Q - K\sigma$$

$$\frac{d\sigma}{dt} = \frac{Q}{V} - \frac{K}{V}.\alpha$$

The general solution of this equation is,

$$\sigma = B + be^{-\beta t} \qquad (4)$$

When t = 0, σ = 0

From equation (4),

$$0 = B + b$$

$$b = \ -B$$

$$\sigma = \left[B - e^{-\beta t}\right]$$

Substituting the values of B and β

$$\sigma = \frac{4Q}{A\alpha v}\left[1 - \frac{e^{-A\alpha t}}{4v}\right] \qquad (5)$$

Equation (5) represents the rise of average sound energy per unit time from the time the source commences to produce sound. The maximum value of average energy per unit volume

$$\sigma_{max} = \frac{4Q}{A\alpha v}$$

Similarly, after the source ceases to emit sound, the decay of the average energy per unit volume is given by

$$\sigma = \frac{4Q}{A\alpha v}e^{-\frac{A\alpha v}{4V}.t}$$

$$\sigma = \sigma_{max}e^{-\frac{A\alpha v}{4V}.t} \qquad (6)$$

The factor $\frac{A\alpha v}{4V}$ gives the reverberation time in the auditorium. If σ_0 represents the minimum audible intensity after a time t_1, then from equation (6)

$$\sigma = \sigma_{max} e^{-\frac{A\alpha v}{4V}.t_1} \quad (7)$$

Here t_1 is the time interval between the cutting off the sound and the time at which intensity falls below the minimum audible level.

From equation (7),

$$\sigma = \sigma_{max} e^{+\frac{A\alpha v}{4V}.t_1}$$

Taking logarithms

$$log_e\left(\frac{\sigma_{max}}{\sigma}\right) = \frac{A\alpha v}{4V}.t_1 \quad (8)$$

Here α and σ_0 change with the frequency of sound.

From calculating the reverberation time, a standard steady intensity is required. Sabine took the value of $\frac{\sigma_{max}}{\sigma} = 10^6$

$$log_e(10^6) = \frac{A\alpha v}{4V}.t_1$$

$$2.303 \times 6 = \frac{A\alpha v}{4V}.t_1$$

Taking velocity of sound approximately at room temperature as 350 m/s.

$$2.303 \times 6 = \frac{A \times 350}{4V}.t_1$$

$$t_1 = \frac{2.303 \times 24V}{350A\alpha}$$

$$t_1 = \frac{0.158V}{A\alpha}$$

In general,

$$t_1 = \frac{0.158V}{\Sigma A\alpha}$$

The above equation represents the Sabine's reverberation time formula

According to equation Sabine's reverberation time formula, the reverberation time is

(i) Directly proportional to the volume of the auditorium;

(ii) Inversely proportional to the area of the ceilings, area of the walls etc

(iii) Inversely proportional to the total absorption plus transmission through open surfaces.

It has been experimentally found that the reverberation time of 1.03 seconds is most suitable for all rooms having approximately a volume of less than 350 cubic metres.

To decrease the reverberation time, the walls of the auditorium are usually covered with material having large absorption coefficients. The area of the surfaces of the walls is also increased in good cinema halls to decrease the reverberation time.

5.7 Acoustic intensity

Acoustic intensity of a sound wave is defined as the average power transmitted per unit area I the direction of propagation of the wave. Instantaneous power per unit area is equal to the product of instantaneous pressure and instantaneous particle velocity. Average power per unit area measures the acoustic intensity.

$$I = \frac{1}{T}\int_0^T Pv\,dt$$

$$P = -\rho C^2 \left(\frac{du}{dx}\right)$$

$$u = A\cos(wt - kx)$$

$$v = \frac{du}{dt} = -\omega A \sin(\omega t - kx)$$

$$\frac{du}{dx} == -Ak\sin(\omega t - kx)$$

$$kC = \omega$$

$$P = -\rho CA\omega \sin(\omega t - kx)$$

Here C is the velocity of sound and ρ is the density of the medium.

$$I = \frac{1}{T}\int_0^T [-\rho CA\omega\sin(\omega t - kx)]\,[\omega A\sin(\omega t - kx)]dt$$

$$= \frac{\rho C\omega^2 A^2}{T}\int_0^T \sin(\omega t - kx)^2 dt$$

$$= \frac{\rho C\omega^2 A^2}{T}\int_0^T \left(cos^2 kx\ sin^2 \omega t + sin^2 \omega t\ cos^2 \omega t - \frac{1}{2}\sin 2\omega t \sin 2kx\right) dt$$

$$= \frac{\rho C \omega^2 A^2}{T} \frac{T}{2}$$

$$I = \frac{1}{2}\, \rho C \omega^2 A^2 \qquad (1)$$

$$P = -\rho C A \omega \sin(\omega t - kx)$$

$$P_{max} = -\rho C A \omega$$

Root mean square value of pressure,

$$P_{RMS} = \frac{P_{max}}{\sqrt{2}}$$

From equation (1),

$$I = \frac{\rho^2 C^2 \omega^2 A^2}{2\rho C}$$

$$I = \frac{(P_{max})^2}{2\rho C}$$

$$I = \frac{\frac{P_{max}}{\sqrt{2}}}{2\rho C}$$

$$I = \frac{P_{RMS}^2}{2\rho C}$$

5.8 Factors Affecting the acoustic of buildings

Reverberation is one of the important single factors that affect the acoustics of a room or a hall. Besides reverberations there are also other factors like as loudness, focusing, echelon effect, extraneous noise, resonance etc.

Loudness

The speech of a person in a hall can be heard by an audience consisting of about 1000 persons. However, to ensure uniform distribution of sound intensity in the hall, electrically amplified loud speakers are used. These speakers are kept at different places in the auditorium and are located generally at a height higher than the speaker's head. Amplifiers, however, make the low frequency tones more prominent and hence the amplification has to be kept low. The presence of low artificial ceilings improves the audibility in general.

Focusing

The presence of cylindrical or spherical surfaces on the walls or the ceiling gives rise to undesirable focusing. In the following figure 5.3 the observer at O receives sound from the speaker along the direct path SO. The observer also receives the sound waves after reflection from the ceiling. Thus the intensity of sound at O is comparatively higher than other positions in the auditorium. It may also happen that the direct and the reflected waves are in opposite phase. This results in minimum intensity of sound at O. Further, the direct and the reflected waves may form a stationary wave pattern. This causes uneven distribution of sound intensity.

Echelon effect

If there is regular structure similar to a flight of stairs or a set of railings in the hall, the sound produced in front of such a structure may produce a musical note due to regular successive echoes of sound reaching the observer. Such an effect is called echelon effect. If the frequency of this note is within the audible range, the listener will hear only this note prominently. To avoid echelon effect, the stair cases are covered with carpets to avoid reflection of sound.

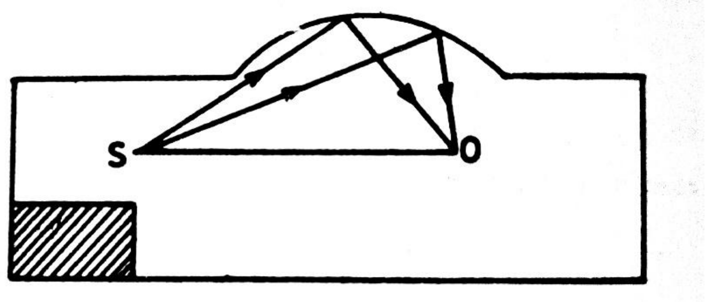


Figure 5. 3

Extraneous noise

The extraneous noise may be due to

- sound received from outside the room
- the sound produced by fans etc., inside the auditorium.

The external sound cannot be completely eliminated but can be minimized by using double or triple windows and doors. Proper attention must also be paid to maximum permissible speed of fans and the rate of air circulation in the room. The air conditioning pipes should be covered with cork and insulated acoustically from the main building.

Resonance

The acoustics of a building may also be affected by resonance. If there is resonance for any audio frequency note, the intensity of the note will be entirely different from the intensity desired. In halls of large size, the resonance frequency is much below the audible limit and harmful effects due to resonance will not be present.

5.9 Piezoelectric crystal method

In 1880, the Curie brothers, discovered that certain asymmetric crystals exhibit Piezo-electric effect. In this effect, if one pair of opposite faces of a crystal is subjected to pressure, the other pair of opposite faces develop opposite electric charges. The sign of the charges changes when the faces are subjected to tension instead of pressure. The converse of Piezo-electric effect is also true. According to this, if alternating voltages are applied to one pair of faces, the corresponding changes in the dimensions of the other pair of faces of the crystal are produced. These changes have been

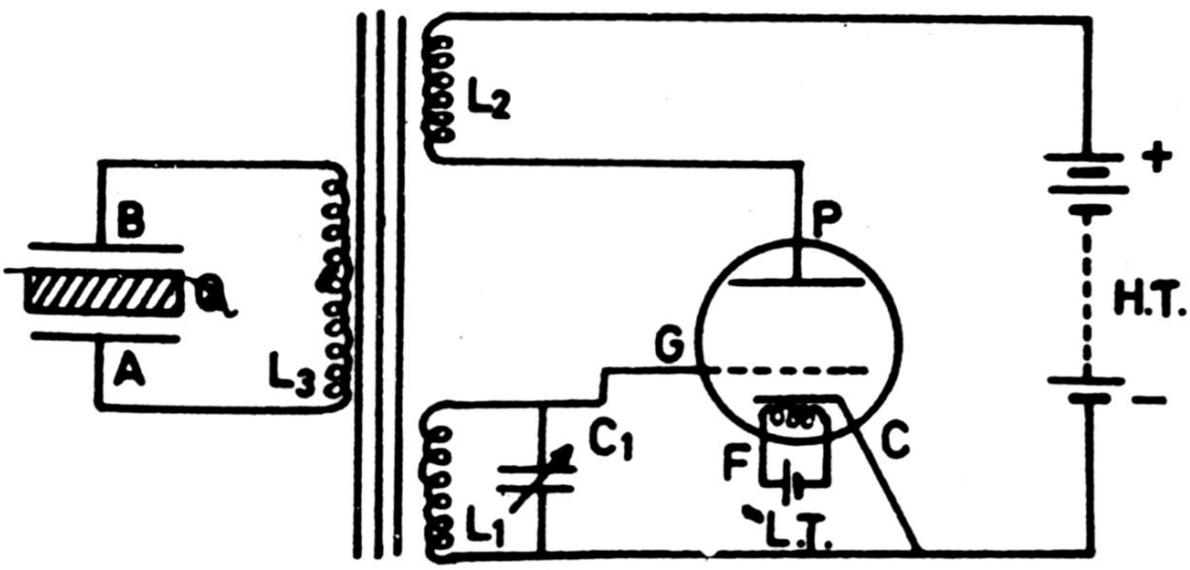


Figure 5. 4

found to be more pronounced when thin slices of crystals of quartz, tourmaline and Rochelle salt are used. The direction of the cut of the crystal with reference to the optic axis is quite important.

Thus, when the two opposite faces of a quartz crystal, their faces being cut perpendicular to the optic axis, are subjected to alternating voltage, the other pair of opposite faces experiences stresses and strains. The quartz crystal will continuously contract and expand. Elastic vibrations are set up in the crystal.

When the frequency of the alternating voltage is equal to the natural frequency of vibration of the crystal or its simple higher multiples, the crystal is thrown into resonant vibrations and the amplitude will be large. These vibrations are longitudinal in nature. The frequency of vibration is

$$n = \frac{p}{2l}\sqrt{\frac{Y}{\rho}}$$

P = 1, 2, 3, … etc

Here Y is the elasticity and ρ is the density of the crystal.

The veity of longitudinal waves in the crystal

$$v = \sqrt{\frac{Y}{\rho}}$$

The value of V = 5.5×10^3 m/s for quartz

For a crystal of length 0.05m, the frequency for the first mode of vibration will be

$$\frac{1 \times 5.5 \times 10^3}{2 \times 0.05} = 5.5 \times 10^4\ hertz$$

The other modes of frequency are simple integral multiples of 5.5×10^4 m/s hertz

For experimental arrangement, the circuit diagram is shown in figure 5.4. *Q* is a thin slice of quartz crystal cut with its opposite faces perpendicular to the optic axis. The crystal is places between two metal plates A and B. The plates A and B are connected to the coil L_3. Coils L_1, L_2 and L_3 are inductively couples. Coil L_2 is connected in the plate circuit. The tank circuit L_1 and C_1 is connected between the grid and the cathode.

The variable condenser C_1 is adjusted so that the frequency of the oscillatory circuit is equal to the natural frequency of one of the modes of vibration of the crystal. This will ensure resonant mechanical vibration in the crystal due to the linear expansion and contraction. Ultrasonic frequencies as high as 5×10^8 hertz can be obtained with this arrangement.

5.10 Magnetostriction effect

Ultrasonic waves can be produced by using the principle of magnetostriction. According to this principle, if a ferromagnetic material in the form of a bar like iron or nickel is subjected to an alternating magnetic field, the bar expands and contracts in length alternately. The frequency of contraction or expansion is twice the frequency of the alternating magnetic field. The alternating magnetic field is produced with the help of an oscillatory circuit. Due to the longitudinal contraction and expansion of the bar, longitudinal compressional waves are produced in the medium surrounding the bar. Frequencies ranging from a few hundred hertz to about 300,000 hertz can be produced with this arrangement.

The experimental arrangement is shown in Figure 5.5. *XY* is a bar of ferromagnetic material say of iron or nickel. The bar is clamped in the middle. L_1 and L_2 are two coils surrounding the bar *PQ*. L_1 and C_1 are connected in parallel and the combination is connected in the plate circuit. L_2 is connected between the grid and the cathode. The milli ammeter is connected in the plate circuit. The values of L_1 and C_1 determine the frequency of the oscillatory circuit.

Initially the bar is magnetized by passing direct current. The condenser C_1 is adjusted so that the frequency of the oscillatory circuit is the same as the natural frequency of longitudinal vibrations of the bar. In this case, the oscillations are maintained by the coupling effect between the coils L_1 and L_2. Any change in the plate current brings about a change in magnetisation and consequently the length of the rod changes. This gives rise to the change in flux in the coil L_2 in the grid circuit. An

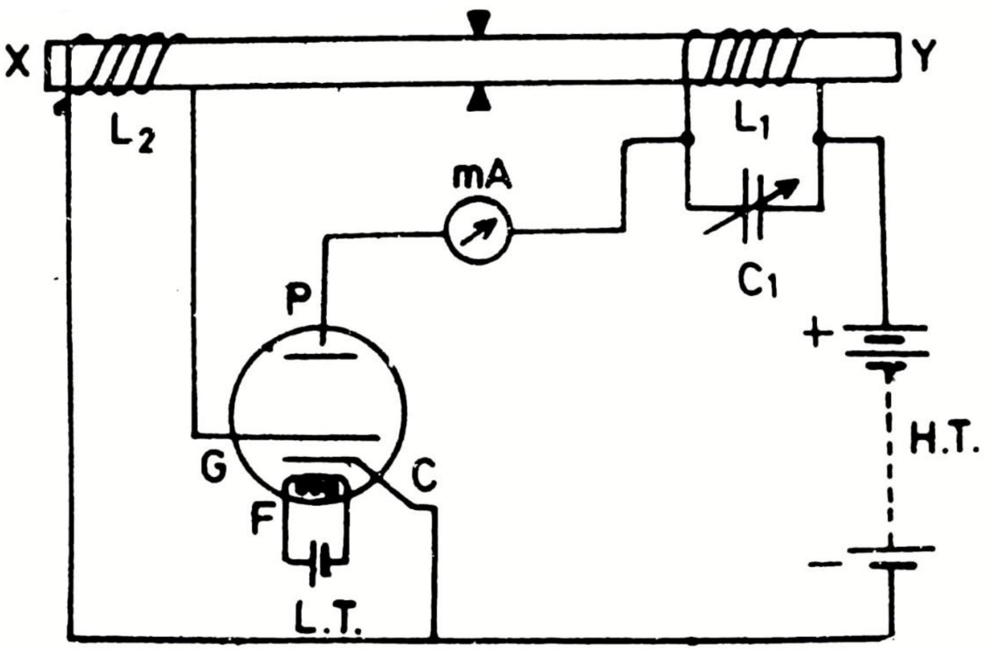


Figure 5. 5

induced e.m.f. is produced in the coil L_2 and this e.m.f. is amplified by the triode valve and reacts back on the coil L_1. In this way, the oscillations are maintained and the amplitude of oscillations will be large.

At the start, the milliammeter shows alternation when the oscillations are set up. The condenser C_1 is adjusted to the position where these alternations are maximum. In this position, the vibrations are most intense.

The frequency of the ultrasonic waves produced by this method will depend on the length, density and elasticity of the material of the bar.

5.11 Application of ultrasonic waves

Depth of Sea:

Ultrasonic waves of high frequency are used to determine the depth of the sea. A Piezo-electric quartz oscillator is used for this purpose. The crystal is placed between two metal plates and the plates are connected to a spark oscillator, producing damped oscillations. The frequency of the damped oscillator is tuned to be the same as the natural frequency of the quartz crystal. The quartz crystal itself acts as a transmitter and a receiver of the ultrasonic waves. The ultrasonic waves transmitted by the crystal are directed towards the bed of the sea. These waves are reflected back from the bed and the echo is detected by the crystal itself. In this case, the metal plates are automatically connected to an amplifier and a canoed ray oscillograph. The time interval between the emitted signal and the echo is determined with the help of the oscillograph. Knowing the velocity of sound through sea water and the time interval, the depth of the sea can be calculated. Suppose, t is the time interval between the transmission of the ultrasonic wave and receipt of the echo and the velocity of sound waves through sea water, then depth of the sea,

$$h = \frac{v \times t}{2}$$

This method is also suitable to detect the presence and depth of submarines, rocks etc., from the surface of sea water. The instrument directly calibrated to show the depth of sea is called a fathometer or echo meter.

Signalling:

Ultrasonic waves are used for directional signaling. The frequency of ultrasonic waves is higher than the audible sound waves. There- fore, the wavelength is comparatively small. Due to the small wavelength, ultrasonic waves can be sent in the form of a short beam. If a quartz crystal, taken in the form a disc of radius r, is used as a source of ultrasonic waves, the angle of the cone containing these waves is given by

$$\sin\theta = \frac{0.61\,\lambda}{r}$$

For small wavelengths, e is small. Even for a small amplitude of the vibrating crystal, large amount of energy is radiated whereas it is not possible in the case of audio frequency waves. Recently, ultrasonic microscope has been invented. It is used to detect concealed objects. The frequency is very high so that the wave length is of the order of the wavelength of visible light.

Heating effects

When a beam of ultrasonic waves is passed through a substance, it gets heated. If ultrasonic waves pass through water at 0°C, water can be made to boil.

Mechanical effects

Ultrasonic drills are used to bore holes in steel and other metals or their alloys. Here the drill oscillates with ultrasonic frequency and can bore any hard metal.

Cracks in metals

Ultrasonic waves can be used to detect cracks or discontinuity in metal structures. In this case, an emitter and detector of ultrasonic waves are used. Ultrasonic waves from the emitter are directed towards the metal. The reflected beam is detected by the detector. If there is a crack or discontinuity, there will be rise in energy received by the detector, if the emitter and the detector are on the same side. If the emitter and the detector are on the opposite sides of the metal, there will be fall in energy at the regions of cracks or discontinuity.

Formation of alloys

Alloys of uniform composition are obtained by subjecting the constituents to an ultrasonic beam. The two constituents are well mixed up by ultrasonic waves, even though the constituents differ in density.

Chemical effect

Ultrasonic waves act like catalytic agents and accelerate chemical reactions. They bring about a number of chemical changes. Some of the chemical applications are as follows:

- When potassium iodide is subjected to ultrasonic waves, it liberates iodine.
- Water is decomposed into hydrogen and hydroxyl ions, by the action of ultrasonic waves.
- Ultrasonic waves reduce mercuric chloride into mercurous chloride.
- Water and oil are immiscible. An emulsion of water and oil is obtained when the mixture is subjected to ultrasonic waves. Similarly, an emulsion of water and mercury can be prepared.
- Ultrasonic waves accelerate crystallization.
- Ultrasonic waves explode nitrogen iodide.

Soldering

Aluminum cannot be soldered by ordinary soldering method. To solder aluminium, ultrasonic waves are used in addition to the electrical soldering iron. The ultrasonic waves remove the oxide film and facilitates soldering.

Medical applications

Ultrasonic waves have a large number of applications in the field of medicine. Some of the important applications are as follows:

- Neuralgic pain. Ultrasonic waves are useful for relieving neuralgic and rheumatic pains. The affected portion of the body is exposed to ultrasonic waves. The waves produce a soothing massage action and relieves pain.
- Arthritis. Ultrasonic waves are used to relieve pain due to arthritis. Here a small metal head, vibrating with a frequency of more than 10° hertz is moved over the skin of the patient. These vibrations after passing through the tissues, produce a deep massage action. The patient is relieved of the pain.
- Contracted fingers. Ultrasonic waves are used to restore the contracted fingers. They are also used to loosen up the scar tissues in various parts of the human body.
- Broken teeth. Ultrasonic waves are used by dentists for the proper extraction of broken teeth.
- Bloodless surgery. Ultrasonic waves are used in bloodless surgery. Here the ultrasonic waves are focused on a sharp instrument and the tissues are destroyed without any loss of blood. American doctors have used such instruments for conducting bloodless brain operations.

Sterilization

Ultrasonic waves can destroy unicellular organisms. is Bacteria perishing under the action of ultrasonic waves. Ultrasonic waves are used in the sterilization of water and milk.

Enemy of lower life.

When some lower animals like rats, frogs, fish, etc., are exposed to ultrasonic waves, they become lame.

Ultrasonic waves are having more and more practical applications in all fields. Active research work is still in progress to study the effect of ultrasonic waves in mechanical, biological, chemical, physical and industrial fields.